Name _____

Mathematics A-Level Induction

Refresher Course to Help with the Transition from GCSE to A-Level Maths

Chiltern Hills Academy
Mathematics Department

Contents

How to use this booklet

Thank you for choosing to study Mathematics in the sixth form at Chiltern Hills Academy. At the end of year 13, you will sit two papers in Pure Mathematics as well as one papers in application. If you have chosen to study Further Mathematics there are various options. However, you will study Pure and Application. The Mathematics Department is committed to ensuring that you make good progress throughout your A-level course. In order that you make the best possible start to the course, we have prepared this booklet.

The material in this refresher course has been designed to enable you to prepare for the demands of A-Level maths. When your course starts in September you will find that your ability to get the most from lessons, and to understand new material, depends crucially upon both having a good facility with algebraic manipulation and undertaking plenty of independent study. We think that this is so important that we are providing you with this workbook to complete before you start the course in September.

It is ***vitally important*** that you spend some time working through the questions in this booklet over the summer - you will need to have a good knowledge of these topics ***before*** you commence the course in September. A-Level maths is a demanding course and good skills in the areas covered within this booklet will be paramount to your success.

You will most-likely have met all the topics before at GCSE. At the start of each section the relevant MyMaths lessons have been indicated; use these to help you if you are stuck with anything or unsure of how to proceed. You should attempt at least the majority of questions in each exercise – not necessarily every question, but enough to ensure you understand the topic thoroughly. The answers are given at the back of the booklet and you should mark your work. If you get questions wrong you should revise the material and try again until you are getting the majority of questions correct. You may find some questions towards the end of each section quite tricky – this is ok, but still give them a try!

In September, you will need to bring your work, and submit it to your teacher who will not mark it but rather will consider if an appropriate amount of work has been completed. **You will also be tested at the start (2nd week) of September to check how well you understand these topics**, so it is important that you have worked through the booklet before then. If you do not achieve a good enough score in this test (70%), a letter of concern will be sent home and you will be re-tested in September (3rd week). A mock test is provided at the back of this booklet, with answers. We hope that you will use this introduction to give you a good start to your A level work and that it will help you enjoy and benefit from the course more.

Mr R Vaddiraju
Assistant Vice-Principal
Director of Mathematics

1. Arithmetic of Fractions

MyMaths.co.uk **Number > Fractions > Adding Subtracting, Multiplying, Dividing**

1. Express each of the following as a fraction in its simplest form. For example $\frac{3}{21}$ can be written as $\frac{1}{7}$. Remember, no calculators!

a) $\frac{20}{45}$ b) $\frac{16}{36}$ c) $-\frac{42}{21}$ d) $\frac{18}{16}$ e) $\frac{30}{30}$ f) $\frac{17}{21}$ g) $-\frac{49}{35}$ h) $\frac{90}{30}$

2. Calculate

a) $\frac{1}{2}+\frac{1}{3}$ b) $\frac{1}{2}-\frac{1}{3}$ c) $\frac{2}{3}+\frac{3}{4}$ d) $\frac{5}{6}-\frac{2}{3}$ e) $\frac{8}{9}+\frac{1}{5}+\frac{1}{6}$ f) $\frac{4}{5}+\frac{3}{7}-\frac{9}{10}$

3. Evaluate the following, expressing each answer in its simplest form.

a) $\frac{4}{5}\times\frac{3}{16}$ b) $2\times3\times\frac{1}{4}$ c) $\frac{3}{4}\times\frac{3}{4}$ d) $\frac{4}{9}\times6$ e) $\frac{15}{16}\times\frac{4}{5}$ f) $\frac{9}{5}\times\frac{1}{3}\times\frac{15}{27}$

4. Evaluate

a) $3 \div \frac{1}{2}$ b) $\frac{1}{2} \div \frac{1}{4}$ c) $\frac{6}{7} \div \frac{16}{21}$ d) $\frac{\frac{3}{4}}{4}$ e) $5 \div \frac{10}{9}$ f) $\frac{3}{4} \div \frac{4}{3}$

5. Express the following as mixed fractions. A mixed fraction has a whole number part and a fractional part. For example, $\frac{13}{5}$ can be written as the mixed fraction $2\frac{3}{5}$.

a) $\frac{5}{2}$ b) $\frac{7}{3}$ c) $-\frac{11}{4}$ d) $\frac{6}{5}$ e) $\frac{12}{5}$ f) $\frac{18}{7}$ g) $\frac{16}{3}$ h) $\frac{83}{9}$

6. Express the following as improper fractions. An improper fraction is 'top-heavy'. Its numerator is greater than its denominator. For example, the mixed fraction $13\frac{4}{5}$ can be written as the improper fraction $\frac{69}{5}$.

a) $2\frac{1}{4}$ b) $3\frac{1}{2}$ c) $5\frac{2}{3}$ d) $-3\frac{2}{5}$ e) $11\frac{4}{6}$ f) $8\frac{2}{9}$ g) $16\frac{3}{4}$ h) $89\frac{2}{7}$

2. Rules and Manipulation of Indices

MyMaths.co.uk **Number > Powers and Roots > Indices 1, 2 and 3**

1. Simplify the following algebraic expressions.

a) $x^3 \times x^4$ b) $y^2 \times y^3 \times y^5$ c) $z^3 \times z^2 \times z$ d) $t^2 \times t^{10} \times t$

e) $a \times a \times a^2$ f) $t^3 t^4$ g) $b^6 b^3 b$ h) $z^7 z^7$

2. Simplify

a) $\frac{x^6}{x^2}$ b) $\frac{y^{14}}{y^{10}}$ c) $\frac{t^{16}}{t^{12}}$ d) $\frac{z^{10}}{z^9}$ e) $\frac{v^7}{v^0}$ f) x^7/x^4

3. Simplify the following:

a) $\frac{10^7}{10^6}$ b) $\frac{10^{19}}{10^{16}}$ c) $\frac{x^7}{x^{14}}$ d) $\frac{x^7}{y^4}$

e) $\frac{(ab)^4}{a^2b^2}$ f) $\frac{9^9 10^{10}}{10^9}$ g) $\frac{x^9y^8}{y^7x^6}$, h) $\frac{(abc)^3}{(abc)^2}$

4. Write the following expressions using only *positive* indices. For example $\dfrac{x^{-4}}{x^{-2}}$ can be written as $\dfrac{1}{x^2}$.

a) $x^{-2}x^{-1}$ b) $\dfrac{3x}{x^{-4}}$ c) $\dfrac{t^{-2}}{t^{-3}}$

d) $(2a^2b^3)(6a^{-3}b^{-5})$ e) $\dfrac{x^{-3}}{5^{-2}}$ f) $\dfrac{(27)^{-1}x^{-1}}{y^{-2}}$

5. Without using a calculator, evaluate

a) $\dfrac{3}{4^{-2}}$ b) 4×3^{-2} c) $3^{-1}9^2(27)^{-1}$

d) $(0.25)^{-1}$ e) $(0.2)^{-2}$ f) $(0.1)^{-3}$

6. Simplify

a) $t^{-6}t^3$ b) $(-y^{-2})(-y^{-1})$ c) $\dfrac{3y^{-2}}{6y^{-3}}$ d) $(-2t^{-1})(-3t^{-2})(-4t^{-3})$

e) $\dfrac{3t^{-2}}{6t^3}$ f) $\dfrac{(2t^{-1})^3}{6t^2}$ g) $\dfrac{(-2t)^3}{(-4t)^2}$

7. Write the following expressions using a *single* index. For example $(5^3)^{-4}$ can be written as 5^{-12}.

a) $(5^3)^5$ b) $(3^3)^3$ c) $(17^2)^4$ d) $(y^3)^6$ e) $(\frac{y^{-1}}{y^{-2}})^3$

f) $(\frac{t^{-2}}{t^4})^3$ g) $(k^{-2})^{-6}$ h) $((-1)^4)^3$ i) $((-1)^{-4})^{-3}$

8. Without the use of a calculator, evaluate

a) $(4^{-1})^2$ b) $(2^2)^{-1}$ c) $(3^2)^2$ d) $(6^{-2})^{-1}$

e) $\left(\frac{2}{5^2}\right)^{-1}$ f) $(-2)^{-1}$ g) $\left(-\frac{2}{3}\right)^{-2}$

9. Write the following expressions without using brackets.

a) $(4^2 5^3)^3$ b) $\left(\frac{3ab}{c^3}\right)^2$ c) $\left(\frac{4^{-2}a^{-3}}{b^{-1}}\right)^2$ d) $(2a^2b)^3$

e) $(3xy^2z^3)^2$ f) $\left(\frac{6}{ab^2}\right)^2$ g) $\left(-\frac{3}{x^2}\right)^2$ h) $\left(\frac{2z^2}{3t}\right)^3$

i) $(-2x)^2$ j) $(-2x^2)^{-2}$ k) $\left(-\frac{2}{x^2}\right)^{-3}$

10. Write the following expressions without using brackets.

a) $(6^{1/2})^3$ b) $(5^{1/3})^6$ c) $(10^{0.6})^4$ d) $(x^2)^{1/3}$

e) $(2x^2)^{1/3}$ f) $(a \times a^2)^{1/2}$ g) $(ab^2)^{1/2}$

11. Write the following expressions without using brackets.

a) $(4^3)^{-1/2}$ b) $(3^{-1/2})^{-1/2}$ c) $(7^{2/3})^4$ d) $(19^{3/2})^{1/3}$

e) $(a^2b^{-3})^{-\frac{3}{2}}$ f) $\left(\dfrac{k^{-1.5}}{\sqrt{k}}\right)^{-2}$

12. Write the following expressions without using brackets.

a) $(5b)^{1/6}$ b) $(3\sqrt{x})^3$ c) $3(\sqrt{x})^3$ d) $(\sqrt{3x})^3$

13. Simplify

a) $x^{1/2}x^{1/3}$ b) $\dfrac{x^{1/2}}{x^{1/3}}$ c) $(x^{1/2})^{1/3}$ d) $(8x^3)^{1/3}$

e) $\sqrt{25y^2}$ f) $\left(\frac{27}{t^3}\right)^{1/3}$ g) $(16y^4)^{1/4}$ h) $\left(x^{1/4}\,x^{1/2}\right)^4$

i) $\sqrt{a^2a^6}$ j) $\sqrt{\dfrac{a^{-4}}{a^{-1}}}$

3. Expanding Brackets and Factorising

MyMaths.co.uk **Algebra > Algebraic Manipulation > Single Brackets, Brackets, Factorising Linear, Factorising Quadratics 1 and 2**

1. Write the following expressions without using brackets:

a) $2(mn)$ b) $2(m+n)$ c) $a(mn)$ d) $a(m+n)$ e) $a(m-n)$

f) $(am)n$ g) $(a+m)n$ h) $(a-m)n$ i) $5(pq)$ j) $5(p+q)$

k) $5(p-q)$ l) $7(xy)$ m) $7(x+y)$ n) $7(x-y)$ o) $8(2p+q)$

p) $8(2pq)$ q) $8(2p-q)$ r) $5(p-3q)$ s) $5(p+3q)$ t) $5(3pq)$

2. Write the following expressions without using brackets and simplify where possible:

a) $(2+a)(3+b)$ b) $(x+1)(x+2)$ c) $(x+3)(x+3)$ d) $(x+5)(x-3)$

3. Write the following expressions without using brackets:

a) $(7+x)(2+x)$ b) $(9+x)(2+x)$ c) $(x+9)(x-2)$ d) $(x+11)(x-7)$

e) $(x+2)x$ f) $(3x+1)x$ g) $(3x+1)(x+1)$ h) $(3x+1)(2x+1)$

i) $(3x+5)(2x+7)$ j) $(3x+5)(2x-1)$ k) $(5-3x)(x+1)$ l) $(2-x)(1-x)$

4. Rewrite the following expressions without using brackets:

a) $(s+1)(s+5)(s-3)$ b) $(x+y)^3$

5. Factorise

a) $5x+15y$ b) $3x-9y$ c) $2x+12y$ d) $4x+32z+16y$ e) $\frac{1}{2}x+\frac{1}{4}y$

6. Factorise

a) $\frac{1}{3}x + \frac{1}{6}xy$ b) $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$ c) $a^2 - a + \frac{1}{4}$ d) $\frac{1}{x^2} - \frac{2}{x} + 1$

7. Factorise

a) $x^2 + 8x + 7$ b) $x^2 + 6x - 7$ c) $x^2 + 7x + 10$ d) $x^2 - 6x + 9$ e) $x^2 + 5x + 6$.

8. Factorise

a) $2x^2 + 3x + 1$ b) $2x^2 + 4x + 2$ c) $3x^2 - 3x - 6$ d) $5x^2 - 4x - 1$
e) $16x^2 - 1$ f) $-x^2 + 1$ g) $-2x^2 + x + 3$

9. Factorise

a) $x^2 + 9x + 14$ b) $x^2 + 11x + 18$ c) $x^2 + 7x - 18$ d) $x^2 + 4x - 77$

e) $x^2 + 2x$ f) $3x^2 + x$, g) $3x^2 + 4x + 1$ h) $6x^2 + 5x + 1$

i) $6x^2 + 31x + 35$ j) $6x^2 + 7x - 5$ k) $-3x^2 + 2x + 5$ l) $x^2 - 3x + 2$

10. Rewrite the following expressions without using brackets, simplifying where possible:

a) $15 - (7 - x)$ b) $15 - 7(1 - x)$

c) $15 - 7(x - 1)$ d) $(2x - y) - x(1 + y)$

e) $x(a + b) - x(a + 3b)$ f) $2(5a + 3b) + 3(a - 2b)$

g) $-(4a + 5b - 3c) - 2(2a + 3b - 4c)$ h) $2x(x - 5) - x(x - 2) - 3x(x - 5)$

11. Rewrite each of the following expressions without using brackets and simplify where possible

a) $2x-(3y+8x)$, b) $2x+5(x-y-z)$, c) $-(5x-3y)$, d) $5(2x-y)-3(x+2y)$

4. Algebraic Fractions

MyMaths.co.uk **Algebra > Algebraic Manipulation > Cancelling. Adding and Multiplying Algebraic Fractions**

1. Express each of the following as a single fraction.

a) $2 \times \frac{x+y}{3}$ b) $\frac{1}{3} \times 2(x+y)$ c) $\frac{2}{3} \times (x+y)$

2. Simplify

a) $3 \times \frac{x+4}{7}$ b) $\frac{1}{7} \times 3(x+4)$ c) $\frac{3}{7} \times (x+4)$ d) $\frac{x}{y} \times \frac{x+1}{y+1}$

e) $\frac{1}{y} \times \frac{x^2+x}{y+1}$ f) $\frac{\pi d^2}{4} \times \frac{Q}{\pi d^2}$ g) $\frac{Q}{\pi d^2/4}$ h) $\frac{1}{x/y}$

3. Simplify a) $\frac{6/7}{s+3}$ b) $\frac{3/4}{x-1}$ c) $\frac{x-1}{3/4}$

4. Simplify $\frac{3}{x+2} \div \frac{x}{2x+4}$

5. Simplify $\dfrac{5}{2x+1} \div \dfrac{x}{3x-1}$

6. Simplify

a) $\dfrac{x}{4} + \dfrac{x}{7}$ b) $\dfrac{2x}{5} + \dfrac{x}{9}$ c) $\dfrac{2x}{3} - \dfrac{3x}{4}$ d) $\dfrac{x}{x+1} - \dfrac{2}{x+2}$

e) $\dfrac{x+1}{x} + \dfrac{3}{x+2}$ f) $\dfrac{2x+1}{3} - \dfrac{x}{2}$ g) $\dfrac{x+3}{2x+1} - \dfrac{x}{3}$ h) $\dfrac{x}{4} - \dfrac{x}{5}$

7. Simplify

a) $\dfrac{1}{x+2} + \dfrac{2}{x+3}$ b) $\dfrac{2}{x+3} + \dfrac{5}{x+1}$ c) $\dfrac{2}{2x+1} - \dfrac{3}{3x+2}$

d) $\dfrac{x+1}{x+3} + \dfrac{x+4}{x+2}$ e) $\dfrac{x-1}{x-3} + \dfrac{x-1}{(x-3)^2}$

5. Surds

MyMaths.co.uk **Number > Powers and Roots > Surds 1 and Surds 2**

Roots, for example $\sqrt{2}$, $\sqrt{5}$, $\sqrt[3]{6}$ are also known as **surds**. A common cause of error is misuse of expressions involving surds. You should be aware that $\sqrt{ab} = \sqrt{a}\sqrt{b}$ but $\sqrt{a+b}$ is NOT equal to $\sqrt{a} + \sqrt{b}$.

1. It is often possible to write surds in equivalent forms. For example $\sqrt{48}$ can be written $\sqrt{3 \times 16} = \sqrt{3} \times \sqrt{16} = 4\sqrt{3}$.

Write the following in their simplest forms:

a) $\sqrt{180}$ b) $\sqrt{63}$

2. By multiplying numerator and denominator by $\sqrt{2}+1$ show that

$$\frac{1}{\sqrt{2}-1} \quad \text{is equivalent to} \quad \sqrt{2}+1$$

3. Simplify, if possible, a) $\sqrt{x^2y^2}$ b) $\sqrt{x^2+y^2}$.

4. Study the following expressions and simplify where possible.

a) $\sqrt{(x+y)^4}$ b) $(\sqrt[3]{x+y})^6$ c) $\sqrt{x^4+y^4}$

5. By considering the expression $(\sqrt{x}+\sqrt{y})^2$ show that

$$\sqrt{x}+\sqrt{y}=\sqrt{x+y+2\sqrt{xy}}$$

Find a corresponding expression for $\sqrt{x}-\sqrt{y}$.

6. Write each of the following as an expression under a single square root sign. (For parts c) and d) see Question 5 above.)

a) $2\sqrt{p}$ b) $\sqrt{p}\sqrt{q^3}$ c) $\sqrt{p}+\sqrt{2q}$ d) $\sqrt{3}-\sqrt{2}$

7. Use indices (powers) to write the following expressions without the root sign.

a) $\sqrt[4]{a^2}$ b) $(\sqrt{3}\times\sqrt{5})^3$

6. Linear Equations

MyMaths.co.uk **Algebra > Equations – Linear > Solving Equations, Equations with Fractions**

In questions $1-35$ solve each equation:

1. $3y - 8 = \frac{1}{2}y$ **2.** $7t - 5 = 4t + 7$ **3.** $3x + 4 = 4x + 3$ **4.** $4 - 3x = 4x + 3$

5. $3x+7 = 7x+2$ **6.** $3(x+7) = 7(x+2)$ **7.** $2x-1 = x-3$ **8.** $2(x+4) = 8$

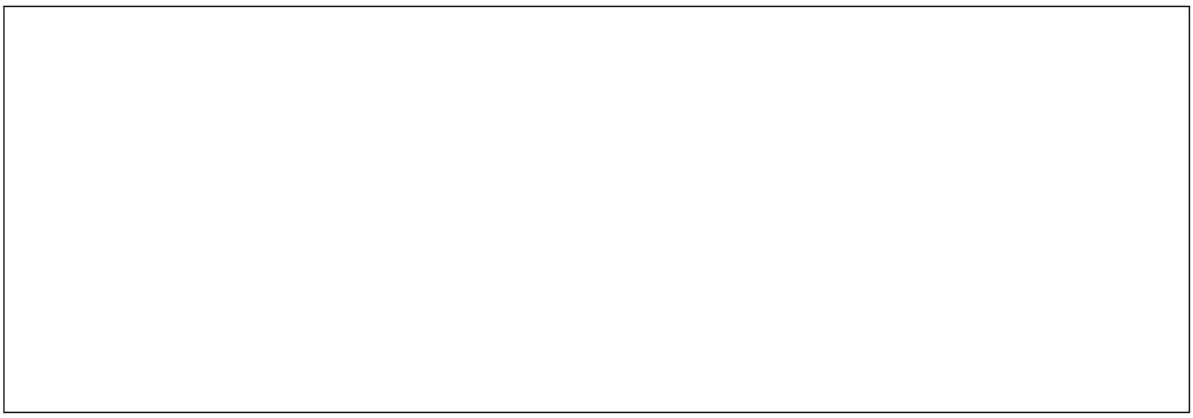
9. $-2(x-3) = 6$ **10.** $-2(x-3) = -6$

11. $-3(3x-1) = 2$ **12.** $2-(2t+1) = 4(t+2)$

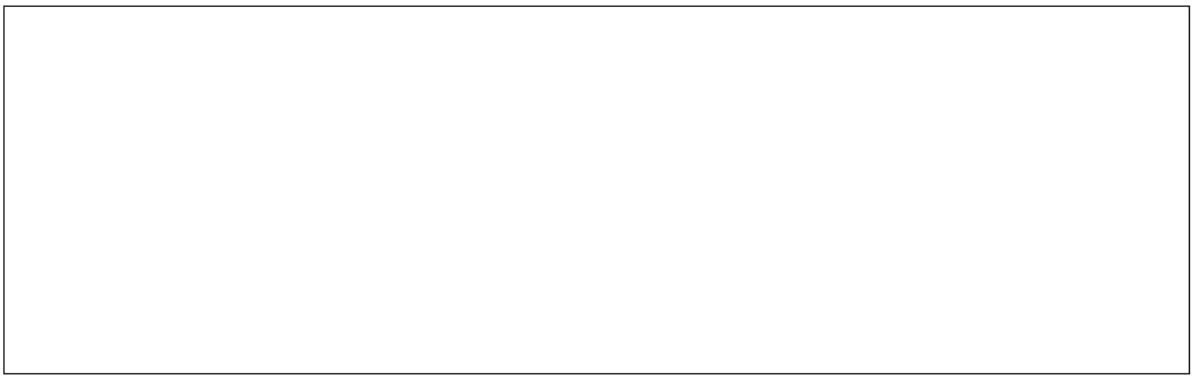
13. $5(m-3) = 8$ **14.** $5m - 3 = 5(m-3) + 2m$

15. $2(y+1) = -8$ **16.** $17(x-2) + 3(x-1) = x$

17. $\frac{1}{3}(x+3) = -9$ **18.** $\frac{3}{m} = 4$ **19.** $\frac{5}{m} = \frac{2}{m+1}$ **20.** $-3x + 3 = 18$

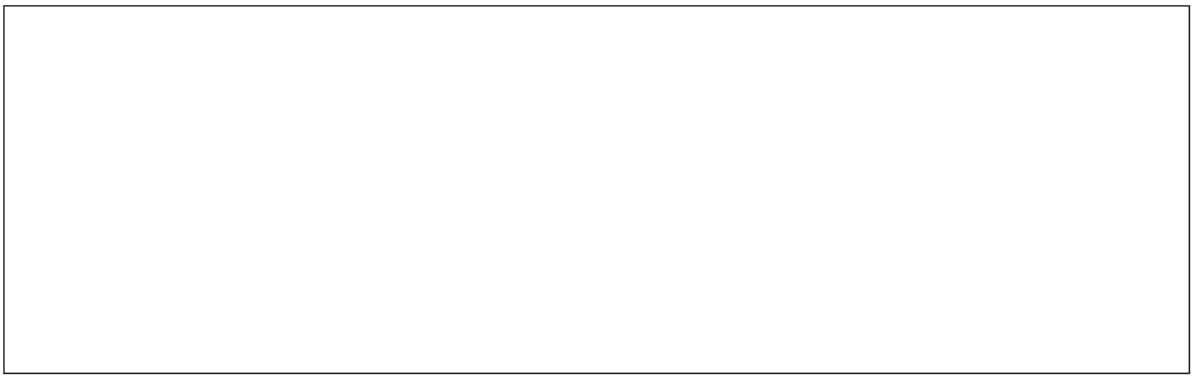


21. $3x + 10 = 31$ **22.** $x + 4 = \sqrt{8}$ **23.** $x - 4 = \sqrt{23}$



24. $\frac{x-5}{2} - \frac{2x-1}{3} = 6$ **25.** $\frac{x}{4} + \frac{3x}{2} - \frac{x}{6} = 1$

26. $\frac{x}{2} + \frac{4x}{3} = 2x - 7$ **27.** $\frac{5}{3m+2} = \frac{2}{m+1}$



28. $\dfrac{2}{3x-2} = \dfrac{5}{x-1}$ **29.** $\dfrac{x-3}{x+1} = 4$ **30.** $\dfrac{x+1}{x-3} = 4$ **31.** $\dfrac{y-3}{y+3} = \dfrac{2}{3}$

32. $\dfrac{4x+5}{6} - \dfrac{2x-1}{3} = x$ **33.** $\dfrac{3}{2s-1} + \dfrac{1}{s+1} = 0$

34. $\dfrac{1}{5x} + \dfrac{1}{4x} = 10.$ **35.** $\dfrac{3}{s-1} = \dfrac{2}{s-5}.$

7. Changing the Subject of a Formulae

MyMaths.co.uk **Algebra > Expressions and Formulae > Rearranging 1 and Rearranging 2**

1. Make t the subject of the formula $p = \dfrac{c}{\sqrt{t}}$.

2. Make N the subject of the formula $L = \dfrac{\mu N^2 A}{\ell}$.

3. In each case make the specified variable the subject of the formula:

a) $h = c + d + 2e$, e b) $S = 2\pi r^2 + 2\pi rh$, h

c) $Q = \sqrt{\dfrac{c+d}{c-d}}$, c d) $\dfrac{x+y}{3} = \dfrac{x-y}{7} + 2$, x

4. Make n the subject of the formula $J = \dfrac{nE}{nL+m}$.

8. Solving Quadratic Equations – Factorising

MyMaths.co.uk **Algebra > Equations – Quadratic > Quadratic Equations**

Solve the following equations by factorisation:

1. $x^2-3x+2=0$ **2.** $x^2-x-2=0$ **3.** $x^2+x-2=0$ **4.** $x^2+3x+2=0$

5. $x^2+8x+7=0$ **6.** $x^2-7x+12=0$ **7.** $x^2-x-20=0$ **8.** $x^2-1=0$

9. $x^2-2x+1=0$ **10.** $x^2+2x+1=0$ **11.** $x^2+11x=0$ **12.** $2x^2+2x=0$

13. $x^2-3x=0$ **14.** $x^2+9x=0$ **15.** $2x^2-5x+2=0$ **16.** $6x^2-x-1=0$

17. $-5x^2+6x-1=0$ **18.** $-x^2+4x-3=0$

9. Solving Quadratic Equations – Completing the Square and Using the Quadratic Formula

MyMaths.co.uk **Algebra > Equations – Quadratic > Completing the Square and Quadratic Formula**

Solve each of the following quadratic equations twice: once by using the formula, then again by completing the square. Obtain your answers in surd, not decimal, form.

1. $x^2 + 8x + 1 = 0$	**2.** $x^2 + 7x - 2 = 0$	**3.** $x^2 + 6x - 2 = 0$
4. $4x^2 + 3x - 2 = 0$	**5.** $2x^2 + 3x - 1 = 0$	**6.** $x^2 + x - 1 = 0$
7. $-x^2 + 3x + 1 = 0$	**8.** $-2x^2 - 3x + 1 = 0$	**9.** $2x^2 + 5x - 3 = 0$
10. $-2s^2 - s + 3 = 0$	**11.** $9x^2 + 16x + 1 = 0$	**12.** $x^2 + 16x + 9 = 0$

13. Show that the roots of $x^2 - 2x + \alpha = 0$ are $x = 1 + \sqrt{1 - \alpha}$ and $x = 1 - \sqrt{1 - \alpha}$.

14. Show that the roots of $x^2 - 2\alpha x + \beta = 0$ are

$$x = \alpha + \sqrt{\alpha^2 - \beta} \quad \text{and} \quad x = \alpha - \sqrt{\alpha^2 - \beta}$$

10. Solving Simultaneous Linear Equations

Algebra > Equations – Simultaneous > Simultaneous 1, 2, 3 and Negative

Solve each of the pairs of simultaneous equations. Where the answer is not a whole number, give your answer as an improper fraction.

1. $4x + 3y = 6$
   $5x - 3y = 21$

2. $3x + 5y = 13$
   $2x + 3y = 8$

3. $x + 4y = 5$
   $4x - 2y = 11$

4. $2a + b = 3$
   $4a - 5b = 20$

5. $5x + 3y = 4$
   $3x + 4y = 9$

6. $6x - 2y = 13$
   $2x + 3y = -3$

7. $3a - 2b = 14$
   $4a + 3b = 13$

8. $5x + 4y = 5$
   $2x + 7y = 29$

9. $6x - 4y = 39$
   $2x + y = 6$

Answers

Section 1. Arithmetic of fractions

1. a) $\frac{4}{9}$, b) $\frac{4}{9}$, c) -2, d) $\frac{9}{8}$, e) 1, f) $\frac{17}{21}$, g) $-\frac{7}{5}$, h) 3
2. a) $\frac{5}{6}$, b) $\frac{1}{6}$, c) $\frac{17}{12}$, d) $\frac{1}{6}$, e) $\frac{113}{90}$, f) $\frac{23}{70}$
3. a) $\frac{3}{20}$, b) $\frac{3}{2}$, c) $\frac{9}{16}$, d) $\frac{8}{3}$, e) $\frac{3}{4}$, f) $\frac{1}{3}$.
4. a) 6, b) 2, c) $\frac{9}{8}$, d) $\frac{3}{16}$, e) $\frac{9}{2}$, f) $\frac{9}{16}$
5. a) $2\frac{1}{2}$, b) $2\frac{1}{3}$, c) $-2\frac{3}{4}$, d) $1\frac{1}{5}$, e) $2\frac{2}{5}$, f) $2\frac{4}{7}$, g) $5\frac{1}{3}$, h) $9\frac{2}{9}$
6. a) $\frac{9}{4}$, b) $\frac{7}{2}$, c) $\frac{17}{3}$, d) $-\frac{17}{5}$, e) $\frac{35}{3}$, f) $\frac{74}{9}$, g) $\frac{67}{4}$, h) $\frac{625}{7}$

Section 2. Manipulation of expressions involving indices

1. a) x^7, b) y^{10}, c) z^6, d) t^{13}, e) a^4, f) t^7, g) b^{10}, h) z^{14}.
2. a) x^4, b) y^4, c) t^4, d) z, e) v^7, f) x^3
3. a) 10, b) 10^3, c) x^{-7}, d) $\frac{x^7}{y^4}$, e) a^2b^2, f) $9^9 \cdot 10$, g) x^3y, h) abc
4. a) $\frac{1}{x^3}$, b) $3x^5$, c) t, d) $\frac{12}{ab^2}$, e) $\frac{5^2}{x^3}$, f) $\frac{y^2}{27x}$
5. a) 48, b) $\frac{4}{9}$, c) 1, d) 4, e) 25, f) 1000
6. a) t^{-3}, b) y^{-3}, c) $\frac{1}{2}y$, d) $-24t^{-6}$, e) $\frac{1}{2t^5}$, f) $\frac{4t^{-5}}{3}$, g) $-\frac{t}{2}$.
7. a) 5^{15}, b) 3^9, c) 17^8, d) y^{18}, e) y^3, f) t^{-18}, g) k^{12}, h) $(-1)^{12} = 1$, i) $(-1)^{12} = 1$.
8. a) $\frac{1}{16}$, b) $\frac{1}{4}$, c) 81, d) 36, e) $\frac{25}{2}$, f) $-\frac{1}{2}$, g) $\frac{9}{4}$
9. a) $4^6 5^9$, b) $\frac{9a^2b^2}{c^6}$, c) $\frac{4^{-4}a^{-6}}{b^{-2}} = \frac{b^2}{4^4a^6}$ d) $8a^6b^3$, e) $9x^2y^4z^6$, f) $\frac{36}{a^2b^4}$, g) $\frac{9}{x^4}$, h) $\frac{8z^6}{27t^3}$, (i) $4x^2$, j) $\frac{1}{4x^4}$, k) $-\frac{x^6}{8}$.
10. a) $6^{3/2}$, b) 5^2, c) $10^{2.4}$, d) $x^{2/3}$, e) $2^{1/3}x^{2/3}$, f) $a^{3/2}$, g) $a^{1/2}b$.
11. a) $4^{-3/2}$, b) $3^{1/4}$, c) $7^{8/3}$, d) $19^{1/2}$, e) $a^{-3}b^{9/2}$, f) k^4.
12. a) $5^{1/6}b^{1/6}$, b) $27x^{3/2}$, c) $3x^{3/2}$, d) $3^{3/2}x^{3/2}$
13. a) $x^{5/6}$, b) $x^{1/6}$, c) $x^{1/6}$, d) $2x$, e) $5y$, f) $\frac{3}{t}$, g) $2y$, h) x^3, i) a^4, j) $a^{-3/2}$

Section 3. Removing brackets and factorisation

1. a) $2mn$, b) $2m + 2n$, c) amn, d) $am + an$, e) $am - an$, f) amn, g) $an + mn$, h) $an - mn$, i) $5pq$, j) $5p + 5q$, k) $5p - 5q$, l) $7xy$, m) $7x + 7y$, n) $7x - 7y$, o) $16p + 8q$, p) $16pq$, q) $16p - 8q$, r) $5p - 15q$, s) $5p + 15q$, t) $15pq$
2. a) $6 + 3a + 2b + ab$, b) $x^2 + 3x + 2$, c) $x^2 + 6x + 9$, d) $x^2 + 2x - 15$
3. a) $14 + 9x + x^2$, b) $18 + 11x + x^2$, c) $x^2 + 7x - 18$,
   d) $x^2 + 4x - 77$, e) $x^2 + 2x$, f) $3x^2 + x$, g) $3x^2 + 4x + 1$
   h) $6x^2 + 5x + 1$, i) $6x^2 + 31x + 35$, j) $6x^2 + 7x - 5$
   k) $-3x^2 + 2x + 5$, l) $x^2 - 3x + 2$
4. a) $s^3 + 3s^2 - 13s - 15$, b) $x^3 + 3x^2y + 3xy^2 + y^3$

5. a) $5(x+3y)$, b) $3(x-3y)$, c) $2(x+6y)$, d) $4(x+8z+4y)$, e) $\frac{1}{2}(x+\frac{1}{2}y)$

6. a) $\frac{x}{3}\left(1+\frac{y}{2}\right)$, b) $\frac{\pi r^2}{3}(2r+h)$, c) $\left(a-\frac{1}{2}\right)^2$, d) $\left(\frac{1}{x}-1\right)^2$.

7. a) $(x+7)(x+1)$, b) $(x+7)(x-1)$, c) $(x+2)(x+5)$, d) $(x-3)(x-3)=(x-3)^2$, e) $(x+3)(x+2)$

8. a) $(2x+1)(x+1)$, b) $2(x+1)^2$, c) $3(x+1)(x-2)$, d) $(5x+1)(x-1)$, e) $(4x+1)(4x-1)$, f) $(x+1)(1-x)$, g) $(x+1)(3-2x)$

9. a) $(7+x)(2+x)$, b) $(9+x)(2+x)$, c) $(x+9)(x-2)$, d) $(x+11)(x-7)$, e) $(x+2)x$, f) $(3x+1)x$, g) $(3x+1)(x+1)$, h) $(3x+1)(2x+1)$ i) $(3x+5)(2x+7)$, j) $(3x+5)(2x-1)$, k) $(5-3x)(x+1)$ l) $(2-x)(1-x)$

10. a) $8+x$, b) $8+7x$, c) $22-7x$, d) $x-y-xy$, e) $-2bx$, f) $13a$, g) $-8a-11b+11c$, h) $7x-2x^2$.

11. a) $-3y-6x$, b) $7x-5y-5z$, c) $-5x+3y$, d) $7x-11y$.

Section 4. Arithmetic of Algebraic Fractions

1. a) $\frac{2(x+y)}{3}$, b) $\frac{2(x+y)}{3}$, c) $\frac{2(x+y)}{3}$

2. a) $\frac{3(x+4)}{7}$, b) $\frac{3(x+4)}{7}$, c) $\frac{3(x+4)}{7}$, d) $\frac{x(x+1)}{y(y+1)}$, e) $\frac{x(x+1)}{y(y+1)}$, f) $Q/4$, g) $\frac{4Q}{\pi d^2}$, h) $\frac{y}{x}$.

3. a) $\frac{6}{7(s+3)}$, b) $\frac{3}{4(x-1)}$, c) $\frac{4(x-1)}{3}$.

4. $\frac{6}{x}$. 5. $\frac{5(3x-1)}{x(2x+1)}$

6. a) $\frac{11x}{28}$, b) $\frac{23x}{45}$, c) $-\frac{x}{12}$, d) $\frac{x^2-2}{(x+1)(x+2)}$, e) $\frac{x^2+6x+2}{x(x+2)}$, f) $\frac{x+2}{6}$, g) $\frac{9+2x-2x^2}{3(2x+1)}$, h) $\frac{x}{20}$.

7. a) $\frac{3x+7}{(x+2)(x+3)}$, b) $\frac{7x+17}{(x+3)(x+1)}$, c) $\frac{1}{(2x+1)(3x+2)}$, d) $\frac{2x^2+10x+14}{(x+3)(x+2)}$, e) $\frac{x^2-3x+2}{(x-3)^2}$

Section 5. Surds

1. a) $\sqrt{180}=\sqrt{36\times 5}=6\sqrt{5}$. b) $\sqrt{63}=\sqrt{9\times 7}=3\sqrt{7}$.

3. a) xy. b) Note that $\sqrt{x^2+y^2}$ is NOT equal to $x+y$.

4. a) $(x+y)^2$, b) $(x+y)^2$, c) $\sqrt{x^4+y^4}$ is NOT equal to x^2+y^2.

5. $\sqrt{x}-\sqrt{y}=\sqrt{x+y-2\sqrt{xy}}$.

6. a) $\sqrt{4p}$, b) $\sqrt{pq^3}$, c) $\sqrt{p+2q+2\sqrt{2pq}}$, d) $\sqrt{5-2\sqrt{6}}$.

7. a) $a^{1/2}$, b) $15^{3/2}$

Section 6. Solving linear equations

1. 16/5,	2. 4,	3. 1,	4. 1/7,
5. 5/4,	6. 7/4,	7. −2,	8. 0,
9. 0,	10. 6,	11. 1/9,	12. −7/6,
13. 23/5,	14. 6,	15. −5,	16. 37/19,
17. −30,	18. 3/4,	19. −5/3,	20. −5,
21. 7,	22. $\sqrt{8}-4$,	23. $\sqrt{23}+4$,	24. −49,
25. 12/19,	26. 42,	27. 1,	28. 8/13,
29. −7/3,	30. 13/3,	31. 15,	32. 7/6,
33. −2/5,	34. $x = 9/200$.	35. $s = 13$.	

Section 7. Transposition of formulae

1. $t = \frac{c^2}{p^2}$, 2. $N = \sqrt{\frac{L\ell}{\mu A}}$,
3. a) $e = \frac{h-c-d}{2}$, b) $h = \frac{S-2\pi r^2}{2\pi r}$, c) $c = \frac{d(1+Q^2)}{Q^2-1}$ d) $x = \frac{21-5y}{2}$.
4. $n = \frac{mJ}{E-LJ}$

Section 8. Solving quadratic equations by factorisation

1. 1,2,	2. $-1, 2$,	3. $-2, 1$,	4. $-1, -2$,
5. $-7, -1$,	6. $4, 3$,	7. $-4, 5$,	8. $1, -1$,
9. 1 twice,	10. −1 twice	11. $-11, 0$,	12. $0, -1$,
13. 0,3,	14. $0, -9$,	15. $2, \frac{1}{2}$,	16. $\frac{1}{2}, -\frac{1}{3}$,
17. $\frac{1}{5}$,1.	18. $1, 3$.		

Section 9. Solving quadratic equations by using a standard formula and by completing the square

Note that answers were requested in surd form. Decimal approximations are not acceptable.

1. $-4 \pm \sqrt{15}$.	2. $-\frac{7}{2} \pm \frac{\sqrt{57}}{2}$.	3. $-3 \pm \sqrt{11}$.	4. $-\frac{3}{8} \pm \frac{\sqrt{41}}{8}$.
5. $-\frac{3}{4} \pm \frac{\sqrt{17}}{4}$.	6. $-\frac{1}{2} \pm \frac{\sqrt{5}}{2}$.	7. $\frac{3}{2} \pm \frac{\sqrt{13}}{2}$.	8. $-\frac{3}{4} \pm \frac{\sqrt{17}}{4}$.
9. $\frac{1}{2}, -3$.	10. $-3/2, 1$.	11. $-\frac{8}{9} \pm \frac{\sqrt{55}}{9}$.	12. $-8 \pm \sqrt{55}$.

Section 10. Solving simultaneous linear equations

1. *x = 3,*	*y = -2*	4. *a = 5/2,*	*y = -2*	7. *a = 4,*	*b = -1*
2. *x = 1,*	*y = 2*	5. *x = -1,*	*y = 3*	8. *x = -3,*	*y = 5*
3. *x = 3,*	*y = ½*	6. *x = 3/2,*	*y = -2*	9. *x = 9/2,*	*y = -3*

Mock Assessment

The test you will sit in September will ask questions similar to this one. Be sure you are able to answer these questions well! You should also be able to complete this test within 1 hour.

You may NOT use a calculator

Useful Formulae If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

1. Expand and simplify

   (a) $(x + 3)(x - 1)$ (b) $(a + 3)^2$ (c) $(2x + 3)(x - 4)$

2. Factorise

   (a) $x^2 - 7x$ (b) $y^2 - 64$ (c) $x^2 - 5x + 6$ (d) $6t^2 - 13t + 5$

3. Simplify (a) $\frac{4x^3y}{8x^2y^3}$ (b) $\frac{3x+2}{3} + \frac{4x-1}{6}$

4. Solve the following equations

   (a) $\frac{h-1}{4} + \frac{3h}{5} = 4$ (b) $x^2 - 8x = 0$ (c) $p^2 + 4p - 12 = 0$

5. Write each of the following as single powers of x and/or y

   (a) $y^3 \times y^2$ (b) $\frac{1}{x^4}$ (c) $(x^2y)^3$ (d) $\frac{x^5}{x^{-2}}$

6. Work out the values of the following, giving your answers as fractions

   (a) 4^{-2} (b) 10^0 (c) $\left(\frac{8}{27}\right)^{\frac{1}{3}}$

7. Solve the simultaneous equations
   $3x - 5y = -11$
   $5x - 2y = 7$

8. Rearrange the following equations to make x the subject

   (a) $v^2 = u^2 + 2ax$ (b) $V = \frac{1}{3}\pi x^2 h$ (c) $y = \frac{x+2}{x+1}$

9. Solve $5x^2 - x - 1 = 0$ giving your solutions in surd form

Mock Assessment – Answers

1) a) $x^2 + 2x - 3$ b) $a^2 + 6a + 9$ c) $2x^2 - 5x - 12$

2) a) $x(x - 7)$ b) $(y + 8)(y - 8)$ c) $(x - 2)(x - 3)$ d) $(3t - 5)(2t - 1)$

3) a) $\frac{x}{2y^2}$ b) $\frac{10x + 3}{6}$

4) a) h = 5 b) $x = 0$ or $x = 8$ c) p = -6 or p = 2

5) a) y^5 b) x^{-4} c) x^6y^3 d) x^7

6) a) $\frac{1}{16}$ b) 1 c) $\frac{2}{3}$

7) $x = 3, y = 4$

8) a) $x = \frac{v^2 - u^2}{2a}$ b) $x = \sqrt{\frac{3V}{\pi h}}$ c) $x = \frac{2 - y}{y - 1}$

9) $x = \frac{1 \pm \sqrt{21}}{10}$