ESTIMATION

What do I need to be able to do?

You should be able to :

- Round numbers to an appropriate accuracy
- Truncate numbers to an appropriate accuracy
- Use inequality notation to identify the error interval due to rounding
- Estimate the value of a calculation

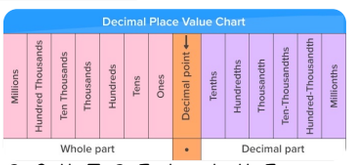
HIGHER TIER ONLY

- Find the greatest and least possible value of a calculation

Key Words

- Significant Figure: the digits in a number which are significant to the size of the number
- Error Interval: The range of values a number could have taken before rounding
- Estimation: finding a number close to the right answer
- Lower Bound: the smallest number that would round up to the estimated value
- Upper Bound: the smallest value that would round up to the next estimated value

Place Value



3 2 4 7 3 5 1 • 1 4 5

"Three million, two hundred and forty seven thousand, three hundred and fifty one point one four five"

Round to Powers of 10

2745 to the nearest 10

If the number is half way between both, we round up

2745 to the nearest 100

Look at the digit in the 'tens column', here we have a 4. Therefore we round down to 2700

2745 to the nearest 1000

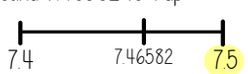
Look at the digit in the 'hundreds column', here we have a 7. Therefore we round up to 3000

Round to Decimal Places

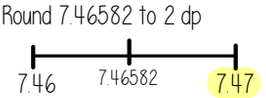
"To 1 dp" means to one number after the decimal
"To 2 dp" means to two numbers after the decimal

Method 1

Round 7.46582 to 1 dp



Round 7.46582 to 2 dp



Method 2

Round 7.46582 to 1 dp

7.4|6582 6, so we round up to 7.5

Round 7.46582 to 2 dp

7.46|582 5, so we round up to 7.47

Round to Significant Figures

Start counting as soon as you get to a non-zero digit

Rounding to 1 significant figure (1 sf)

Round 1394 to 1 sf = 1000
Round 265 to 1 sf = 300
Round 32 to 1 sf = 30
Round 1.87 to 1 sf = 2
Round 0.439 to 1 sf = 0.4
Round 0.008722 to 1 sf = 0.009
Round 0.0005043 to 1 sf = 0.0005

Rounding to 2 significant figures (2 sf)

Round 1394 to 2 sf = 1400
Round 265 to 2 sf = 270
Round 32 to 2 sf = 32
Round 1.87 to 2 sf = 1.9
Round 0.439 to 2 sf = 0.44
Round 0.008722 to 2 sf = 0.0087
Round 0.0005043 to 2 sf = 0.00050

Truncation

Truncate 3.828 to 1 decimal place

3.8|28 We ignore any digits after the first decimal place. So the answer is 3.8

Truncate 3.828 to 2 decimal places

3.82|8 We ignore any digits after the second decimal place. So the answer is 3.82. Notice that if we were to round 2.828 to 2 decimal places, we could get a different answer (3.83)

Truncate 3.828 to 1 significant figure

3|828 We ignore any digits after the first significant figure. So the answer is 3

Truncate 3.828 to 3 significant figures

3.82|8 We ignore any digits after the third significant figure. So the answer is 3.82. Again notice that if we were to round this to 3 sf, the answer would be 3.83!

Truncate 0.0037281 to 3 significant figures

0.00372|81 0.00372

Error Intervals

If a number has been rounded, it is important to consider what possible values the exact value could have been. If we have a puppy that weighs 4kg to the nearest kg, it could actually weigh anything from 3.5kg to 4.5kg! To describe all the possible values that a rounded number could be, we use upper and lower bounds

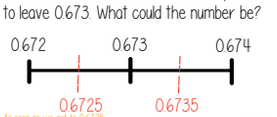
A plant is 35cm tall, rounded to the nearest cm, what was the shortest and tallest height of the plant?

As soon as we get to 34.5, we round up to 35cm so this is the smallest possible value

As soon as we get to 35.5, we round up to 36cm so the largest possible value is 35.49999

34.5cm ⩽ height < 35.5cm

A number was rounded to 3 decimal places to leave 0.673. What could the number be?



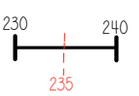
As soon as we get to 0.6725, we round up to 0.673 so this is the smallest possible value

As soon as we get to 0.6735, we round up to 0.674 so the largest possible value is 0.6734999

0.6725 ⩽ x < 0.6735

The speed of a train is 230 km/h truncated to 2 significant figures. What was the range for the true value of the speed?

Anything which starts with 23_ will be truncated to 230



230 ⩽ speed < 240

Estimating Calculations

Estimate the value of 28 x 48

If we round both to 1 sf, this gives;
30 x 50 = 1500

Therefore 28 x 48 ≈ 1500

Estimate the value of (59.3 ÷ 12.09) + 23.4

We can approximate this sum to be (60 ÷ 12) + 20 = 25

Therefore, (59.3 ÷ 12.09) + 23.4 ≈ 25

Estimate the value of $\frac{(4.2 \times 2.4)^2}{\sqrt{5}}$

We can estimate that (4.2 x 2.4) is approximately equal to 4 x 2 (8)

Now to deal with √5! We know that 4 is a square number and it is close to 5 so we can say that √5 is approximately equal to √4 (2).

The sum becomes $\frac{(4 \times 2)^2}{\sqrt{4}} = 32$

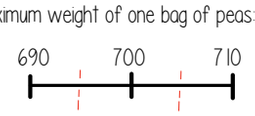
so $\frac{(4.2 \times 2.4)^2}{\sqrt{5}} \approx 32$

Using Error Intervals in Calculations

HIGHER TIER ONLY

A bag of peas has a mass of 700g (to the nearest 10g). Find the maximum mass of 5 bags of peas.

Maximum weight of one bag of peas:



= 705g

Therefore, the maximum weight of 5 bags of peas = 5 x 705 = 3525g

A = 30 (to the nearest whole number)
B = 11.5 (to the nearest 1 decimal place)
C = 300 (to the nearest 1 significant figure)

Error Interval for A: 29.5 < A < 30.5
Error Interval for B: 11.45 < B < 11.55
Error Interval for C: 250 < C < 350

Calculate the maximum value of A + B
UB of A + UB of B : 30.5+11.55 = 42.05

Calculate the minimum value of A x C
LB of A x LB of C: 29.5 x 250 = 7375

Calculate the maximum value of C ÷ B
UB of C ÷ LB of B = 350 ÷ 11.45 = 30.7 (2dp)