

FACTORS, MULTIPLES AND PRIMES

What do I need to be able to do?

You should be able to:

- Understand and use factors
- Understand and use multiples
- Recognise prime numbers
- Recognise square/triangular numbers
- Find common factors, including HCF
- Find common multiples, including LCM
- Express a number as the product of it's prime factors

Key Words

- Multiple: found by multiplying any number by a positive integer
- Factor: integers that multiply together to get another number
- Prime: an integer with only two factors (1 and itself)
- HCF: the highest common factor of two or more numbers
- LCM: the lowest common multiple of two or more numbers
- Product: multiply terms

Factors

A number can have many factors!

Example: what are the factors of 12?

- 1×12
- 2×6
- 3×4

So the factors of 12 are 1, 2, 3, 4, 6, 12

How to find factors

Be systematic! Always find your factor pairs and then write them in ascending order. This way you can be sure you've not missed any out!

Multiples

Eg. What are the multiples of 4?

$4 \times 1, 4 \times 2, 4 \times 3, 4 \times 4$ etc.
4, 8, 12, 16, 20

This list never ends!

'The multiples of a number make up it's times table'

Is 15 a multiple of 3?

5 5 5

As I can share 15 into 3 equally sized parts, 15 is a multiple of 3!

$$3 \times 5 = 15$$

non-EXAMPLE

Why is 10 not a multiple of 4?

$4 \times 2.5 = 10$ but 2.5 is not an integer therefore 10 cannot be a multiple of 4!

Prime Numbers

- Always an integer
- Has only two factors; 1 and itself

Not in any other times tables apart from it's own

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2 is the smallest, and only even, prime number

1 is not a prime number

A prime number has 2 factors, 1 and itself. 1 only has 1 factor (itself/1) therefore it isn't prime!

Square Numbers



They're called square numbers as, when arranged in an array, they make a square!

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225...

Square numbers have an odd number of factors.

Triangular Numbers



They're called triangular numbers as they make a triangle!

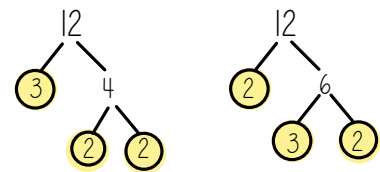
If you add two consecutive triangular numbers, you get a square number!

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120...

Product of Prime Factors

Example 1

Write 12 as a product of it's prime factors

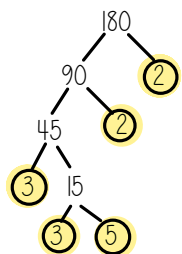


Both of these trees represent the same decomposition

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

Example 2

Write 180 as a product of it's prime factors



$$180 = 2 \times 2 \times 5 \times 3 \times 3 = 2^2 \times 3^2 \times 5$$

Always try to write your final answer in ascending order using index notation!

Using prime factor decomposition

If we know that 12 written as a product of it's prime factors, how does that help us to write 36 as a product of it's prime factors?

We know $12 \times 3 = 36$ therefore we can multiply our answer by three and $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

What about 120?

Well 120 is 10×12 so we can say

$$120 = 2 \times 2 \times 3 \times 10 = 2^3 \times 3 \times 5$$

Remember $10 = 2 \times 5$

Lowest Common Multiple (LCM)

Example 1

What is the LCM of 6 and 8?

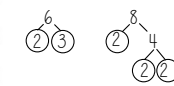
6 - 6, 12, 18, 24, 30
8 - 8, 16, 24, 32, 40

The first time their multiples match is 24 therefore:

the LCM of 6 and 8 is 24

Example 2

What is the LCM of 6 and 8?

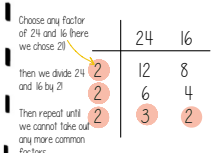


We just multiply all of the numbers in the Venn diagram together to find the LCM!

$$\text{LCM of 6 and 8} = 3 \times 2 \times 2 \times 2 = 24$$

Example 3a

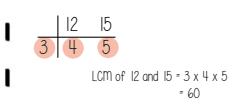
What is the LCM of 24 and 16?



$$\text{LCM of 16 and 24} = 2 \times 2 \times 2 \times 3 \times 2 = 48$$

Example 3b

What is the LCM of 12 and 15?



$$\text{LCM of 12 and 15} = 3 \times 4 \times 5 = 60$$

Highest Common Factor (HCF)

Example 1

What is the HCF of 6 and 8?

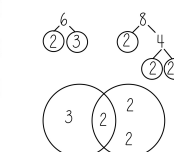
6 - 1, 2, 3, 6
8 - 1, 2, 4, 8

The biggest number which is a factor of both 6 and 8 is 2, therefore

the HCF of 6 and 8 is 2

Example 2

What is the HCF of 6 and 8?

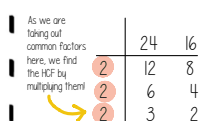


As we are looking for the highest common factor we are looking for the factors which the two numbers share. These can be found in the Venn diagram!

HCF of 6 and 8 = 2

Example 3a

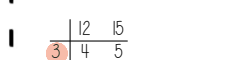
What is the HCF of 24 and 16?



$$\text{HCF of 16 and 24} = 2 \times 2 \times 2 = 8$$

Example 3b

What is the HCF of 12 and 15?



$$\text{HCF of 12 and 15} = 3$$