INDICES

What do I need to be able to do?

You should be able to:

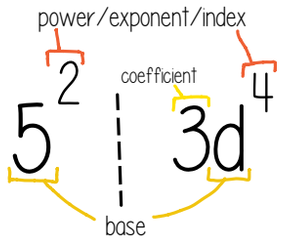
- Add/subtract with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices
- Be familiar with the key results
- Work with negative exponents

HIGHER TIER ONLY

- Work with fractional exponents

Key Words

- Base: the number that gets multiplied by a power
- Power: the number of times the number is used in a multiplication.
- Exponent: power (see above)
- Index: power (see above)
- Coefficient: a number used to multiply a variable
- Variable: a letter which represents an unknown number
- Commutative: changing the order of the operations doesn't change the result



Addition Law for Indices

$$a^m \times a^n = a^{m+n}$$

Examples

- $2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = \underline{2^5}$
- $k^4 \times k^2 = k \times k \times k \times k \times k \times k = \underline{k^6}$

Subtraction Law for Indices

$$a^m \div a^n = a^{m-n}$$

Examples

$5^3 \div 5 = \frac{5 \times 5 \times 5}{5} = \underline{5^2}$

$d^5 \div d^2 = \frac{d \times d \times d \times d \times d}{d \times d} = \underline{d^3}$

Further Examples

Remember it is best practice to write your variables in alphabetical order!

1. $4w \times 5z = 4 \times 5 \times w \times z = \underline{20wz}$

Multiply the coefficients together and then consider the variables!

2. $3a \times 4a^2 \times 2a$
$= 3 \times 4 \times 2 \times a \times a \times a \times a$
$= \underline{24a^4}$

Remember if there is no power written, it is to the power of 1

3. $(t^3)^2 = t^3 \times t^3$
$= t \times t \times t \times t \times t \times t$
$= \underline{t^6}$

Don't forget that if you square something, you multiply it by itself!

4. $3p^2 \times 4p^3 \div 6p^4 = \frac{3p^2 \times 4p^3}{6p^4}$
$= \frac{12p^5}{6p^4}$
$= \underline{2p}$

Don't forget about the order of operations!!

Using the subtraction law, 5-4=1

Spotting Patterns

$2^3 = 2 \times 2 \times 2 = 8$ (+1)

$2^2 = 2 \times 2 = 4$ (×2, ÷2)

$2^1 = 2$ (-1, ÷2)

$2^0 = 1$ (-1, ÷2)

$2^{-1} = \frac{1}{2}$

Each time I add one to the power, I multiply by 2

Each time I take one from the power, I divide by 2

Therefore, 2 to the power of 0 is 1. Remember anything to the power 0 is 1.

If we carry this on, we can even say what 2 to the power of a negative number is!

$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

$2^{-3} = \frac{1}{8} = \frac{1}{2^3}$

We can even spot that 2 to the power of -2 is the same as 1 over 2 to the power of 2 (or 2 squared)

FRACTIONAL INDICES

HIGHER TIER ONLY

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

Examples

$25^{\frac{1}{2}} = \sqrt{25} = \underline{5}$ $\quad 8^{\frac{1}{3}} = \sqrt[3]{8} = \underline{2}$

$$a^{\frac{n}{m}} = (\sqrt[m]{a})^n$$

Examples

$25^{\frac{3}{2}} = (\sqrt[2]{25})^3 = 5^3 = \underline{125}$

Remember that this is the same as $(25^{\frac{1}{2}})^3$

Harder Examples

$(81x^2)^{\frac{1}{2}} = \sqrt{81x^2} = \underline{9x}$

$(9c^4)^{\frac{3}{2}} = (\sqrt{9c^4})^3 = (3c^2)^3 = \underline{27c^6}$

Remember this means we cube EVERYTHING inside the brackets

$(32f^{20})^{\frac{3}{5}} = (\sqrt[5]{32f^{20}})^3 = (2f^4)^3 = \underline{8f^{12}}$

It is really helpful to know the powers of 2,

2
4
8
16
32 ...

Refer to the ladder on the right if you're struggling to spot the patterns!

Square and Cube Numbers

When working with indices, it is helpful to know your square and cube numbers!

SQUARE NUMBERS

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225...

CUBE NUMBERS

1, 8, 27, 81, 125, 216, 343, 512...

You are expected to know these!

NEGATIVE FRACTIONAL INDICES

HIGHER TIER ONLY

EXAMPLE 1

$8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}$

Remember this means the cube root of 8!

EXAMPLE 2

$25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}} = \frac{1}{5^3} = \frac{1}{125}$

Remember this is the same as $(25^{\frac{1}{2}})^3$

EXAMPLE 3

$(343x^9)^{-\frac{2}{3}} \div x^3$
$= \frac{1}{(343x^9)^{\frac{2}{3}}} \div x^3$
$= \frac{1}{(7x^3)^2} \div x^3$
$= \frac{1}{49x^6} \times \frac{1}{x^3} = \frac{1}{49x^9}$

Don't forget the order of operations!

Remember instead of dividing by x cubed, we can multiply by the reciprocal

KEY THINGS TO REMEMBER

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1$$

$$a^{-m} = \frac{1}{a^m}$$

HIGHER TIER ONLY

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$a^{\frac{n}{m}} = (\sqrt[m]{a})^n$$