

YEAR 7
KNOWLEDGE ORGANISERS



BLOCK: ALGEBRAIC THINKING

Sequences

Algebraic notation

Equality and Equivalence

YEAR 7 — ALGEBRAIC THINKING... Sequences

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Describe and continue both linear and non-linear sequences
- Explain term to term rules for linear sequence
- Find missing terms in a linear sequence

Keywords

Sequence: items or numbers put in a pre-decided order

Term: a single number or variable

Position: the place something is located

Rule: instructions that relate two variables

Linear: the difference between terms increases or decreases by the same value each time

Non-linear: the difference between terms increases or decreases in different amounts

Difference: the gap between two terms

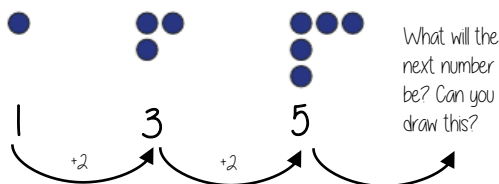
Arithmetic: a sequence where the difference between the terms is constant

Geometric: a sequence where each term is found by multiplying the previous one by a fixed non zero number

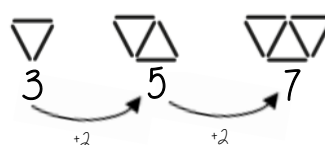


Describe and continue a sequence diagrammatically

Count the number of circles or lines in each image



Predict and check terms



CHECK — draw the next terms



Predictions:

Look at your pattern and consider how it will increase.

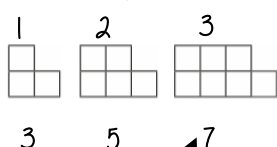
e.g. How many lines in pattern 6?

Prediction - 13

If it is increasing by 2 each time - in 3 more patterns there will be 6 more lines

Sequence in a table and graphically

Position: the place in the sequence



Term: the number or variable (the number of squares in each image)

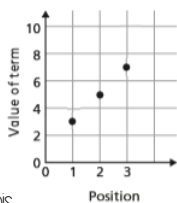
In a table

Position	1	2	3
Term	3	5	7

+2 +2

Because the terms increase by the same addition each time this is **linear** — as seen in the graph

Graphically



Linear and Non Linear Sequences

Linear Sequences — increase by addition or subtraction and the same amount each time

Non-linear Sequences — do not increase by a constant amount — quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

Fibonacci Sequence — look out for this type of sequence

0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms

Continue Linear Sequences

7, 11, 15, 19...

How do I know this is a linear sequence?

It increases by adding 4 to each term

How many terms do I need to make this conclusion?

At least 4 terms — two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?

You continue to repeat the same difference through the next positions in the sequence.

Continue non-linear Sequences

1, 2, 4, 8, 16 ...

How do I know this is a non-linear sequence?

It increases by multiplying the previous term by 2 — this is a geometric sequence because the constant is multiply by 2

How many terms do I need to make this conclusion?

At least 4 terms — two terms only shows one difference not if this difference is constant (a common difference)

How do I continue the sequence?

You continue to repeat the same difference through the next positions in the sequence.

Explain term-to-term rule

How you get from term to term

Try to explain this in full sentences not just with mathematical notation

Use key maths language — doubles, halves, multiply by two, add four to the previous term etc

To explain a whole sequence you need to include a term to begin at...

The next term is found by tripling the previous term
The sequence begins at 4

4, 12, 36, 108...

First term

YEAR 7 — ALGEBRAIC THINKING...

Algebraic notation

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What do I need to be able to do?

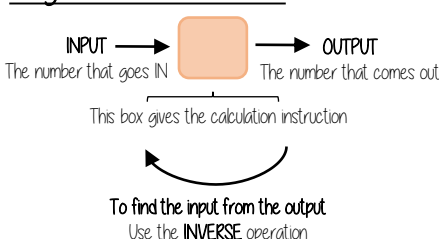
By the end of this unit you should be able to:

- Be able to use inverse operations and "operation families".
- Be able to substitute into single and two step function machines.
- Find functions from expressions.
- Form sequences from expressions.
- Represent functions graphically.

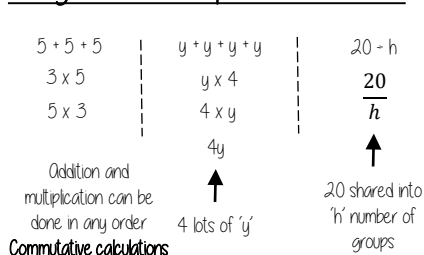
Keywords

- Function:** a relationship that instructs how to get from an input to an output.
- Input:** the number/ symbol put into a function.
- Output:** the number/ expression that comes out of a function.
- Operation:** a mathematical process.
- Inverse:** the operation that undoes what was done by the previous operation. (The opposite operation)
- Commutative:** the order of the operations do not matter.
- Substitute:** replace one variable with a number or new variable.
- Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
- Evaluate:** work out
- Linear:** the difference between terms increases or decreases by the same value each time
- Sequence:** items or numbers put in a pre-decided order

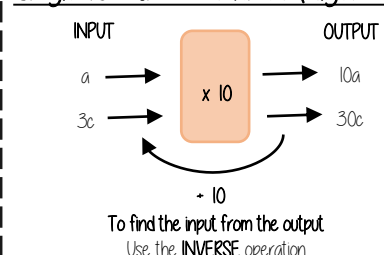
Single function machines



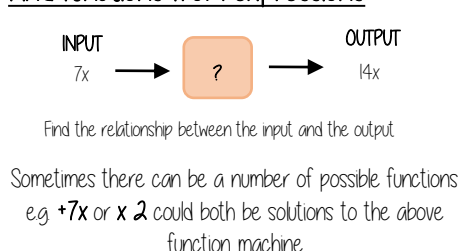
Using letters to represent numbers



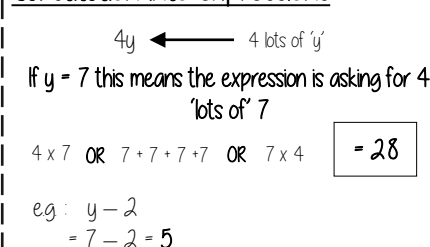
Single function machines (algebra)



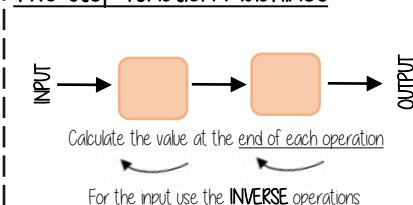
Find functions from expressions



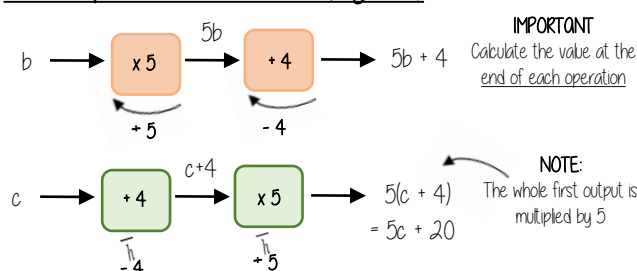
Substitution into expressions



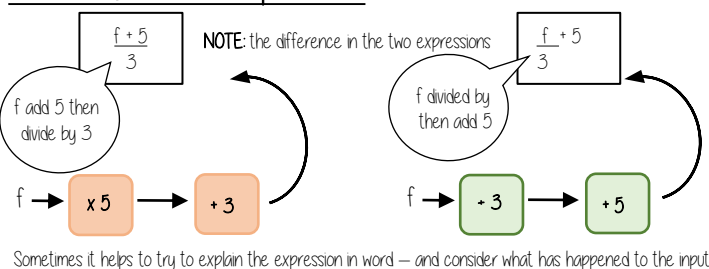
Two step function machines



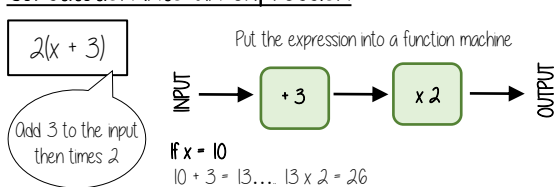
Two step function machines (algebra)



Find functions from expressions

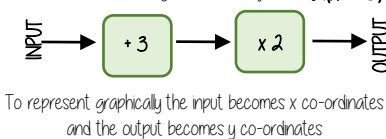


Substitution into an expression



Representing functions graphically

Take the function and generate a sequence $2(x + 3)$

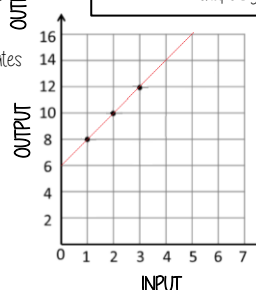


$$y = 2(x + 3)$$

INPUT (x)	1	2	3
OUTPUT (y)	8	10	12

This becomes a co-ordinate pair (2, 10) to plot on a graph

Not all graphs will be linear only those with an integer value for x. Powers and fractions generate differently shaped graphs.



NOTE:
Because this is a linear graph you can predict other values

Forming a sequence

INPUT	1	2	3
OUTPUT	8	10	12

The substitution is the 'input' value
The OUTPUT becomes the sequence

YEAR 7 — ALGEBRAIC THINKING

Equality and Equivalence

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve linear equations
- Understand like and unlike terms
- Simplify algebraic expressions

Keywords

Equality: two expressions that have the same value

Equation: a mathematical statement that two things are equal

Equals: represented by '=' symbol — means the same

Solution: the set or value that satisfies the equation

Solve: to find the solution

Inverse: the operation that undoes what was done by the previous operation (The opposite operation)

Term: a single number or variable

Like: variables that are the same are 'like'

Coefficient: a multiplicative factor in front of a variable e.g. $5x$ (5 is the coefficient, x is the variable)

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Equality

$$2 + 14 = 5 + 5 + 6$$

16

16

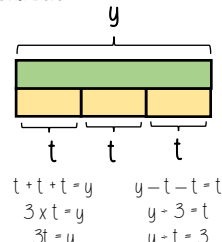
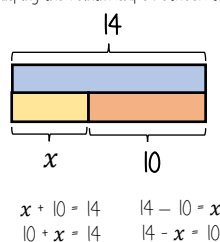
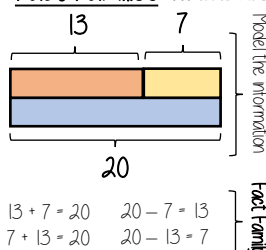
"Is equal to"

Saying it out loud sometimes helps you to understand equality

The sum on the left has the same result as the sum on the right

Fact Families

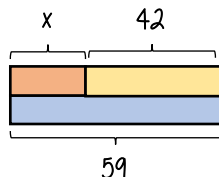
Use a bar model to display the relationships between terms and numbers



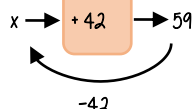
Solve one step equations (+/-)

There is more to this than just spotting the answer

$$x + 42 = 59$$



Don't forget you know how to use function machines



Solve one step equations (x/÷)

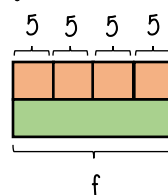
$$\frac{f}{4} = 5$$

$$f - 4 = 5$$

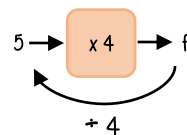
$$f - 5 = 4$$

$$5 \times 4 = f$$

$$4 \times 5 = f$$



Don't forget you know how to use function machines



Like and unlike terms

Like terms are those whose variables are the same

♥ and 3♥ are like terms
 the variable is the same

★ and 3♥ are unlike terms
 the variables are NOT the same

Examples and non-examples

Like terms

y , $7y$
 $2x^2$, x^2
 ab , $10ba$
 5 , -2

Un-like terms

y , $7x$
 $2x^2$, $2c^2$
 ab , $10a$
 5 , $-2t$

Note here ab and ba are commutative operations, so are still like terms

Equivalence

Check equivalence by substitution
 e.g. $m = 10$

$$5m$$

$$5 \times 10$$

$$= 50$$

$$2 \times 2m$$

$$2 \times (2 \times 10)$$

$$= 2 \times 20$$

$$= 40$$

$$7m - 3m$$

$$(7 \times 10) - (3 \times 10)$$

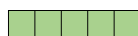
$$= 70 - 30$$

$$= 40$$

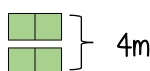
Equivalent expressions

Repeat this with various values for m to check

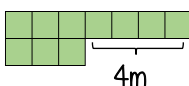
$$5m$$



$$2 \times 2m$$



$$7m - 3m$$



Collecting like terms $\equiv$ symbol

The $\equiv$ symbol means equivalent to

It is used to identify equivalent expressions

Collecting like terms

Only like terms can be combined

$$4x + 5b - 2x + 10b$$

$2x + 15b$

Common misconceptions

$$2x + 3x^2 + 4x \equiv 6x + 3x^2$$

Although they both have the x variable x^2 and x terms are unlike terms so can not be collected

YEAR 7

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BLOCK: PLACE VALUE AND PROPORTION

Place value & ordering integers & decimals

Fraction, decimal & percentage equivalence

YEAR 7 — PLACE VALUE AND PROPORTION...

FDP equivalence

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert fluently between fractions, decimals & percentages

Keywords

Fraction: how many parts of a whole we have

Decimal: a number with a decimal point used to separate ones, tenths, hundredths etc.

Percentage: a proportion of a whole represented as a number between 0 and 100

Place value: the numerical value that a digit has decided by its position in the number

Placeholder: a number that occupies a position to give value

Interval: a range between two numbers

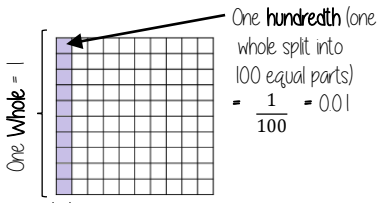
Tenth: one whole split into 10 equal parts

Hundredth: one whole split into 100 equal parts

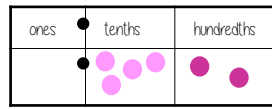
Sector: a part of a circle between two radius (often referred to as looking like a piece of pie)

Recurring: a decimal that repeats in a given pattern

Tenths and hundredths

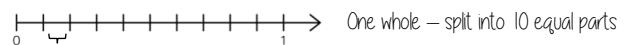


One tenth (one whole split into 10 equal parts) = $\frac{1}{10} = 0.1$

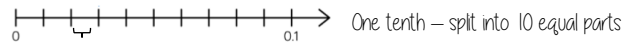


0 ones, 5 tenths and 2 hundredths
 $0 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.01 + 0.01$
 $= 0 + 0.5 + 0.02$
 $= 0.52$

On a number line

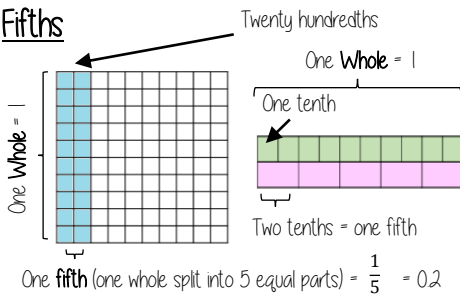


One tenth = $\frac{1}{10} = 0.1$



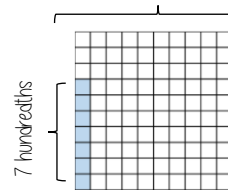
One hundredth = $\frac{1}{100} = 0.01$

Fifths



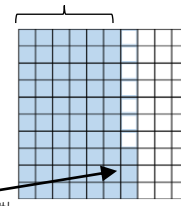
Percentages on a hundred grid

100% = a whole = 100 hundredths



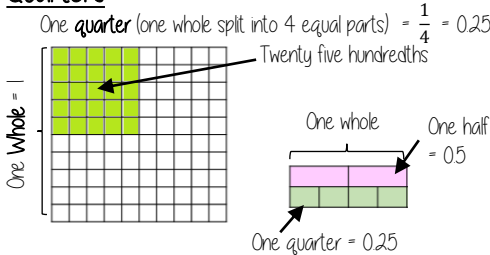
7 hundredths
 7 out of 100
 7%

6 tenths



6 tenths and 3 hundredths
 63 hundredths
 63%

Quarters



Simple pie charts



A pie chart has 360° so all FDP calculations are out of 360

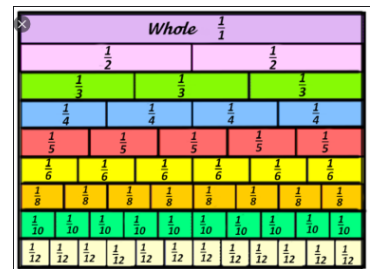
Split into 10 parts
 = 10% = 36°

Split into 2 parts
 = 50% = 180°

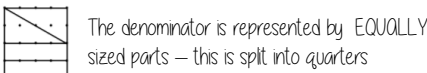
Split into 5 parts
 = 20% = 72°

Equivalent fractions

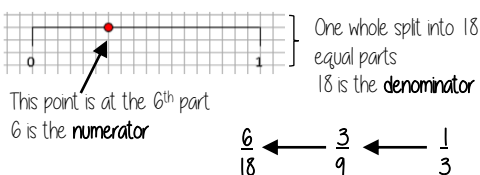
Represent equivalence with fraction walls



Fractions — on a diagram



Fractions — on a number line

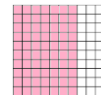


Convert FDP

$\frac{70}{100}$

This also means
 70 = 100

70 out of 100 squares
 70 "hundredths"
 = 7 "tenths"
 0.7



70 hundredths
 = 70%

Using a calculator

$\frac{70}{100}$

S=D

Convert to a decimal

This will give you the answer in the simplest form

$\times 100$ converts to a percentage

Be careful of recurring decimals
 e.g. $\frac{1}{3} = 0.333333$
 $\frac{1}{3} = 0.\dot{3}$
 The dot above the 3

YEAR 7

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BLOCK: APPLICATION OF NUMBER

Solving problems with addition & subtraction

Solving problems with multiplication & division

Fractions & Percentages of amounts

YEAR 7 — APPLICATION OF NUMBER

Solving problems with addition and subtraction

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What do I need to be able to do?

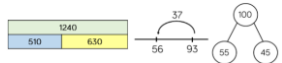
By the end of this unit you should be able to:

- Understand properties of addition/ subtraction
- Use mental strategies for addition/subtraction
- Use formal methods of addition/subtraction for integers
- Use formal methods of addition/subtraction for decimals
- Solve problems in context of perimeter
- Solve problems with finance, tables and timetables
- Solve problems with frequency trees
- Solve problems with bar charts and line charts

Keywords

- Commutative:** changing the order of the operations does not change the result
- Associative:** when you add or multiply you can do so regardless of how the numbers are grouped
- Inverse:** the operation that undoes what was done by the previous operation (The opposite operation)
- Placeholder:** a number that occupies a position to give value
- Perimeter:** the distance/ length around a 2D object
- Polygon:** a 2D shape made with straight lines
- Balance:** in financial questions — the amount of money in a bank account
- Credit:** money that goes into a bank account
- Debit:** money that leaves a bank account

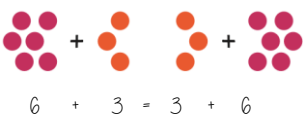
Addition/ Subtraction with integers



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative



The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	4	2	7
-	2	4	9

Remember the place value of each column
You may need to move 10 ones to the ones column to be able to subtract

Addition/ Subtraction with decimals

4	.	3	8
7	.	9	0
			+

0 can be used to fill empty places with value



If represents 1 instead of 100

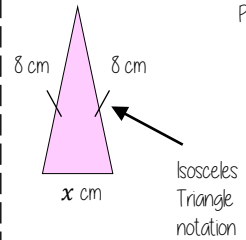
$$5.43 + \frac{8}{10}$$

Revisit Fraction — Decimal equivalence
 $5.43 + 0.8$

The decimal place acts as the placeholder and aligns the other values

Solve problems with perimeter

Perimeter is the length around the outside of a polygon



The triangle has a perimeter of 25cm
Find the length of x
 $8\text{cm} + 8\text{cm} + x\text{cm} = 25\text{cm}$
 $16\text{cm} + x\text{cm} = 25\text{cm}$
 $x\text{cm} = 9\text{cm}$

Solve problems with finance

Profit = Income - Costs

Credit — Money coming into an account

Debit — Money leaving an account

Money uses a two decimal place system
14.2 on a calculator represents £14.20

Check the units of currency — work in the same unit

Tables and timetables

Distance tables

London	Cardiff	Glasgow	Belfast
211			
556	493		
518	392	177	

This shows the distance between Glasgow and London
It is where their row and column intersects

Bus/ Train timetables

Harton	1005	1045	1130
Bridge	1024	1106	1147
Aville	1051	1133	1205
Ware	1117	1202	1233

Each column represents a journey, each row represents the time the 'bus' arrives at that location

TIME CALCULATIONS — use a number line

Two-way tables

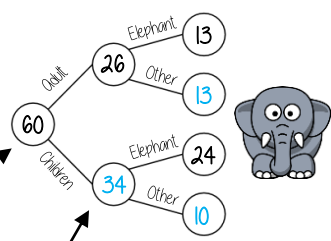
	H	T
H	HH	HT
T	TH	TT

Where rows and columns intersect is the outcome of that action

Frequency trees

60 people visited the zoo one Saturday morning
26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant

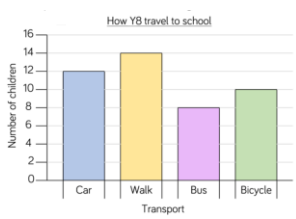
The overall total '60 people'



A frequency tree is made up from part-whole models
One piece of information leads to another

Probabilities or statements can be taken from the completed trees
e.g. 34 children visited the zoo

Bar and line charts



Use addition/ subtraction methods to extract information from bar charts

e.g. Difference between the number of students who walked and took the bus
Walk frequency — bus frequency

When describing changes or making predictions

- Extract information from your data source
- Make comparisons of difference or sum of values
- Put into the context of the scenario

YEAR 7 — APPLICATION OF NUMBER

Solving problems with multiplication and division

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and use factors
- Understand and use multiples
- Multiply/ Divide integers and decimals by powers of 10
- Use formal methods to multiply
- Use formal methods to divide
- Understand and use order of operations
- Solve area problems
- Solve problems using the mean

Keywords

Array: an arrangement of items to represent concepts in rows or columns
Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Mil: prefix meaning one thousandth
Centi: prefix meaning one hundredth
Kilo: prefix meaning multiply by 1000
Quotient: the result of a division
Dividend: the number being divided
Divisor: the number we divide by

Factors

Arrays can help represent factors
 5×2 or 2×5
Factors of 10
1, 2, 5, 10
 10×1 or 1×10
The number itself is always a factor

Square numbers have an ODD number of factors

Factors of 4
1, 2, 4
Factors of 36
1, 2, 3, 4, 6, 9, 12, 18, 36
Be strategic - Lay factors out in pairs can help you not to miss any

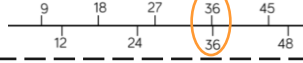
Multiples



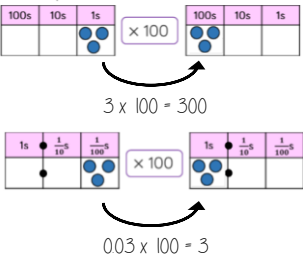
Bar models can represent by something is a multiple. Eg 20 is a multiple of 4

Lowest Common Multiples

LCM of 9 and 12
9: 9, 18, 27, 36, 45, 54
12: 12, 24, 36, 48, 60
The first time their multiples match
LCM = 36



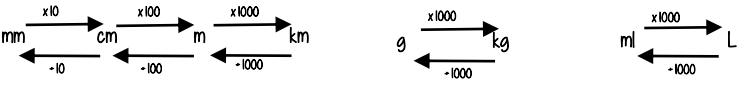
Multiply/ Divide by powers of 10



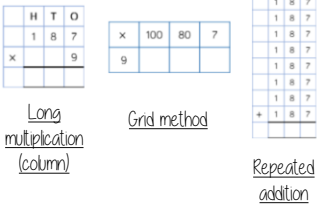
Repeated multiplication and division by powers of 10 is commutative
 $\div 10$ then $\div 10 \rightarrow \div 100$

Metric conversions

Useful Conversions



Multiplication methods



Less effective method especially for bigger multiplication

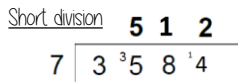
Multiplication with decimals

Perform multiplications as integers
e.g. $0.2 \times 0.3 \rightarrow 2 \times 3$
Make adjustments to your answer to match the question: $0.2 \times 10 = 2$
 $0.3 \times 10 = 3$
Therefore $6 \div 100 = 0.06$

Estimations: Using estimations allows a 'check' if your answer is reasonable

Division methods

$3584 \div 7 = 512$



Complex division
 $\div 24 = \div 6 \div 4$
Break up the divisor using factors

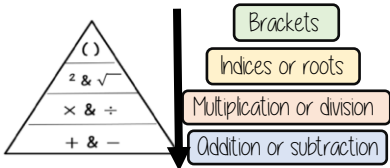
Division with decimals

The placeholder in division methods is essential - the decimal lines up on the dividend and the quotient

$24 \div 0.02 \rightarrow 24 \div 0.2 \rightarrow 240 \div 2$

All give the same solution as represent the same proportion
Multiply the values in proportion until the divisor becomes an integer

Order of operations

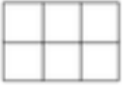


If you have multiple operations from the same tier work from left to right
e.g. $10 - 3 + 5 \rightarrow 10 - 3 \rightarrow 7 + 5$

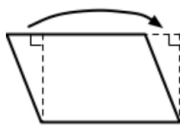
$6 \times 4 + 8 \times 2$
 $24 + 16 = 40$

Area problems

Rectangle
Base x Perpendicular height

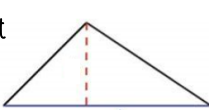


Parallelogram/ Rhombus
Base x Perpendicular height



Triangle
 $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$

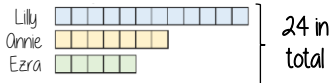
A triangle is half the size of the rectangle it would fit in



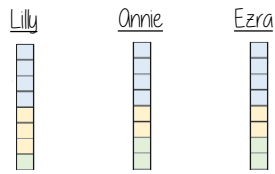
Mean problems

Mean - a measure of average
It gives an idea of the central value

Lilly, Annie and Ezra have the following cubes



Finding the mean amount is the average amount each person would have if shared out equally



The mean number of blocks would be 8 each

YEAR 7 — APPLICATION OF NUMBER

Fractions and percentages of amounts

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Find a fraction of a given amount
- Use a given fraction to find the whole or other fractions
- Find the percentage of an amount using mental methods
- Find the percentage of a given amount using a calculator

Keywords

Fraction: how many parts of a whole we have

Equivalent: of equal value

Whole: a number with no fractional or decimal part

Percentage: parts per 100 (uses the % symbol)

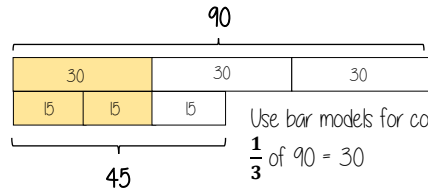
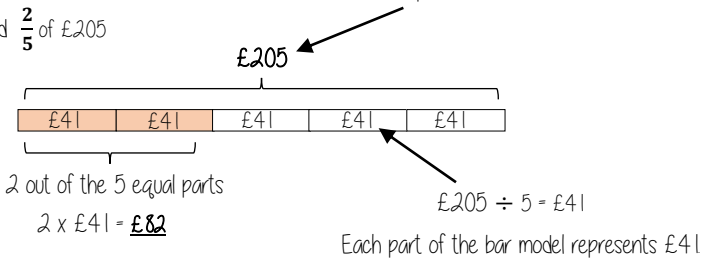
Place Value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Convert: change into an equivalent representation, often fraction to decimal to a percentage cycle.

Fraction of a given amount

Find $\frac{2}{5}$ of £205

The bar represents the whole amount



Use bar models for comparisons

$$\frac{1}{3} \text{ of } 90 = 30$$

$$\frac{2}{3} \text{ of } 45 = 30$$

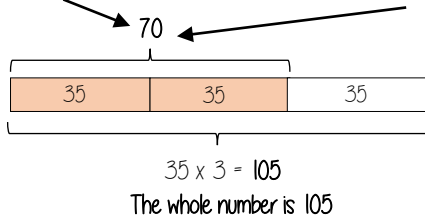
$$\therefore \frac{1}{3} \text{ of } 90 = \frac{2}{3} \text{ of } 45$$

Use a fraction of amount

$\frac{2}{3}$ of a value is 70. What is the whole number?

$$70 \div 2 = 35$$

Each part of the bar model represents 35

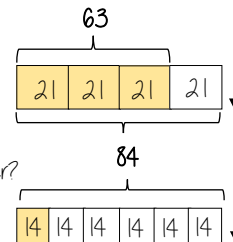


The wording of the question is important to setting up the bar model

$\frac{3}{4}$ of a number is 63.

What is $\frac{1}{6}$ of the number?

$$= 14$$

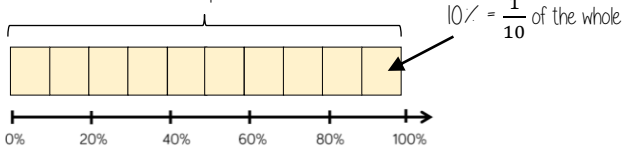


Find the whole

Use the whole to find a given part

Find the percentage of an amount (Mental methods)

The whole represents 100%



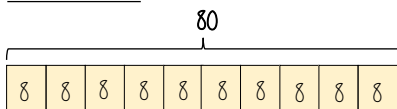
$$10\% = \frac{1}{10} \text{ of the whole}$$

$$50\% = \frac{5}{10} = \frac{1}{2} \text{ of the whole}$$

$$20\% = \frac{2}{10} = \frac{1}{5} \text{ of the whole}$$

$$5\% = \frac{1}{20} \text{ of the whole}$$

Find 65% of 80



Method 1

$$\begin{aligned} 65\% &= 10\% \times 6 + 5\% \\ &= (8 \times 6) + 4 \\ &= 52 \end{aligned}$$

Method 2

$$\begin{aligned} 65\% &= 50\% + 10\% + 5\% \\ &= 40 + 8 + 4 \\ &= 52 \end{aligned}$$

For bigger percentages it is sometimes easier to take away from 100%

Find the percentage of an amount (Calculator methods)



Using a multiplier

Find 65% of 80

Fraction, decimal, percentage conversion

$$65\% = \frac{65}{100} = 0.65$$

The multiplier

$$0.65 \times 80 = 52$$

Using the percent button

Find 65% of 80

This brings up the % button on screen
You will see 65%

Type 65

Press **SHIFT** **C** **(%)**

Press **x** 80 and then press **=**

You can also use the calculator to support non calculator methods and find 1% or 10% then add percentages together

"of" can represent 'x' in calculator methods

YEAR 7
KNOWLEDGE ORGANISERS



BLOCK: DIRECTED NUMBER

Operations with equations and directed
numbers

YEAR 7 — DIRECTED NUMBER

Operations with equations and directed numbers

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Perform calculations that cross zero
- Add/ Subtract directed numbers
- Multiply/ Divide directed numbers
- Evaluate algebraic expressions
- Solve two-step equations
- Use order of operations with directed number

Keywords

Subtract: taking away one number from another.

Negative: a value less than zero.

Commutative: changing the order of the operations does not change the result

Product: multiply terms

Inverse: the opposite function

Square root: a square root of a number is a number when multiplied by itself gives the value (symbol $\sqrt{\quad}$)

Square: a term multiplied by itself.

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Perform calculations that cross zero

Number lines are useful to help you visualise the calculation crossing 0

$4 - 6 = -2$

Use the number line to guide subtraction of 6

Start at 4

Find the difference between 6 and -4

From 6 to 0
6
From 0 to -4
4
10 beads between them

$-5 + 5 = 0$

Rearrangements of the same equation

$5 - 5 = 0$

Add directed numbers

$2 + -4 = -2$

Zero pair $(-1 + 1 = 0)$

Two -1 's left $= -2$

$8 + -3 = 5$

Partitioning

$8 + -3 = 5$

$5 + 3 + -3 = 5$

Partition the value to create a zero pair calculation

Generalisation $+ - = -$

Subtract directed numbers

Representation for calculation

$2 - -1 = 3$

Take away one

Start with the representation of 2

$2 - -3 = 5$

Generalisation $- - = +$

Multiply/ Divide directed numbers

Two representations of the same calculation

$2 \times -3 = -6$

Negative, Negative calculation

-2×-3

This is the negative of 2×-3

$-2 \times -3 = 6$

The act of making counters into their negative is turning them over

Divisions are the inverse operations

Evaluate algebraic expressions

$a = 5$

$b = -4$

$a^2 = 5^2$

$a^2 = 25$

$b^2 = (-4)^2$

$b^2 = 16$

With negative numbers the brackets are important so that it performs -4×-4 .

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$

$3b - 2a = 3(-4) - 2(5) = -12 - 10 = -22$

Two-step equations

Bar Model

$4x + 2 = 10$

$10 - 4x = 2$

Representing the same question (use fact families)

Function machine

$x \rightarrow x4 \rightarrow +2 \rightarrow 10$

Inverse operations to find x

Use order of operations

Brackets

Indices or roots

Multiplication or division

Addition or subtraction

Remember square roots have a positive and negative value

x	-3	-2	-1	0	1	2	3
-3	9	6	3	0	-3	-6	-9
-2	6	4	2	0	-2	-4	-6
-1	3	2	1	0	-1	-2	-3
0	0	0	0	0	0	0	0
1	-3	-2	-1	0	1	2	3
2	-6	-4	-2	0	2	4	6
3	-9	-6	-3	0	3	6	9

YEAR 7
KNOWLEDGE ORGANISERS



BLOCK: FRACTIONAL THINKING

Addition & subtraction of fractions

YEAR 7 — FRACTIONAL THINKING

Addition and subtraction of fractions

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert between mixed numbers and fractions
- Add/Subtract unit fractions (same denominator)
- Add/Subtract fractions (same denominator)
- Add/Subtract fractions from integers
- Use equivalent fractions
- Add/Subtract any fractions
- Add/Subtract improper fractions and mixed numbers
- Use fractions in algebraic contexts

Keywords

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represents the total number of parts

Equivalent: of equal value

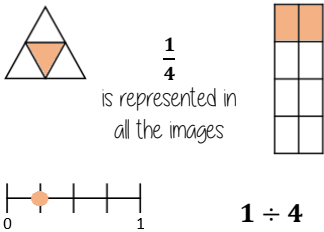
Mixed numbers: a number with an integer and a proper fraction

Improper fractions: a fraction with a bigger numerator than denominator

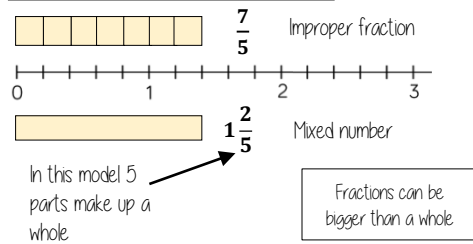
Substitute: replace a variable with a numerical value

Place value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

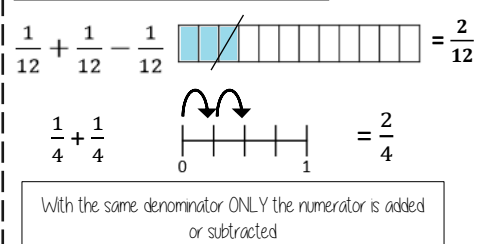
Representing Fractions



Mixed numbers and fractions



Add/Subtract unit fractions

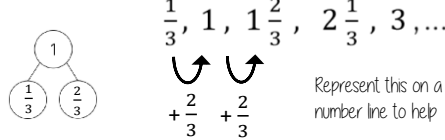


Add/Subtract fractions

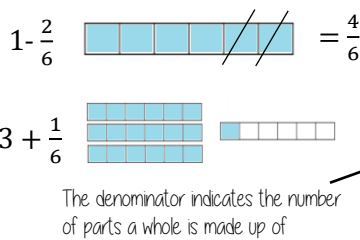
Same denominator

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

Sequences

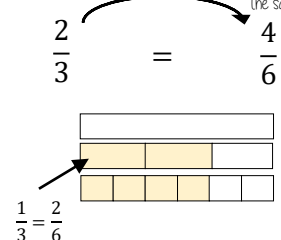


Add/Subtract from integers

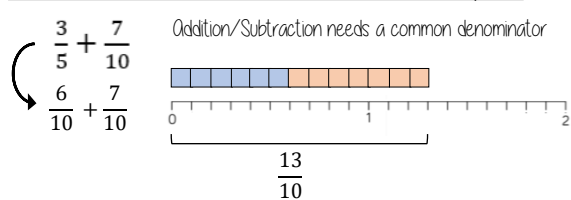


Equivalent fractions

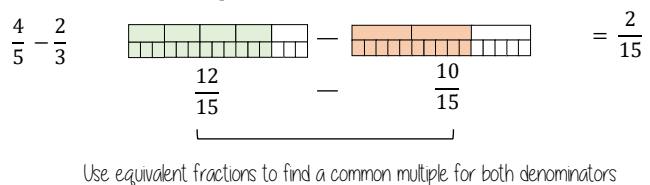
Numerator and denominator have the same multiplier



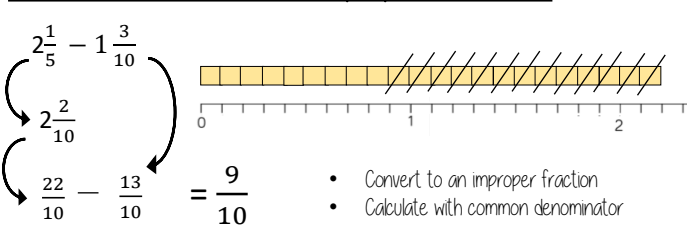
Add/Subtraction fractions (common multiples)



Add/Subtraction any fractions



Add/Subtraction fractions (improper and mixed)



Partitioning method

$$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$$

Fractions in algebraic contexts

$p = 5$ $m = 2$

$$k - \frac{5}{8} = 2$$

Apply inverse operations

$$k = 2 + \frac{5}{8}$$

$$b + \frac{7}{9} = \frac{25}{27}$$

Form expressions with fractions

$$b + \frac{7}{9} \rightarrow b + \frac{7}{9}$$

$$\frac{p}{8} + \frac{1}{m}$$

Substitution

$$\frac{5}{8} + \frac{1}{2}$$

Fractions and decimals

$$\frac{1}{10} = 0.1$$

$$\frac{1}{100} = 0.01$$

Example

$$\frac{6}{10} + 0.3$$

$$\frac{6}{10} + \frac{3}{10}$$

$$0.6 + 0.3$$

Remember to use equivalent fractions and common denominators

YEAR 7
KNOWLEDGE ORGANISERS



BLOCK: LINES AND ANGLES

Constructing, measuring and using
geometric notation
Developing geometric reasoning

YEAR 7 — LINES AND ANGLES

Constructing, measuring and using geometric notation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

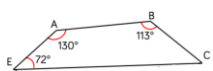
- Use letter and labelling conventions
- Draw and measure line segments and angles
- Identify parallel and perpendicular lines
- Recognise types of triangle
- Recognise types of quadrilateral
- Identify polygons
- Construct triangles (SAS, SSS, ASA)
- Draw Pie charts

Keywords

Polygon: A 2D shape made with straight lines
Scalene triangle: a triangle with all different sides and angles
Isosceles triangle: a triangle with two angles the same size and two angles the same size
Right-angled triangle: a triangle with a right angle
Frequency: the number of times a data value occurs
Sector: part of a circle made by two radii touching the centre
Rotation: turn in a given direction
Protractor: equipment used to measure angles
Compass: equipment used to draw arcs and circles

Letter and labelling convention

The letter in the middle is the angle
 The arc represents the angle

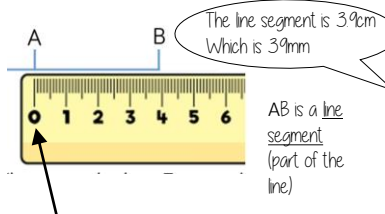


Angle Notation: three letters ABC
 This is the angle at B = 113°

Line Notation: two letters EC
 The line that joins E to C

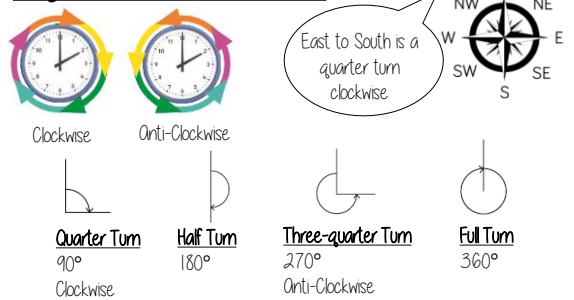
Draw and measure line segments

Conversions: $1\text{cm} = 10\text{mm}$, $1\text{m} = 100\text{cm}$

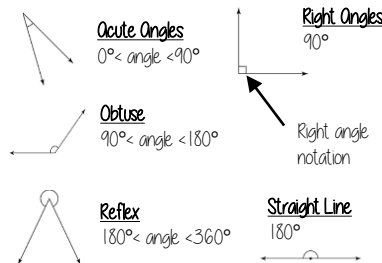


Make sure the start of the line is at 0.

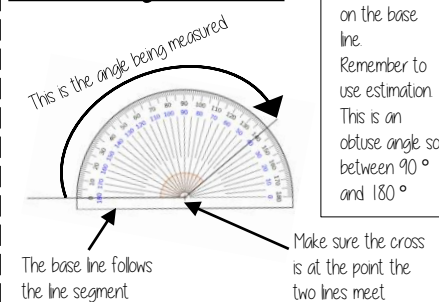
Angles as measures of turn



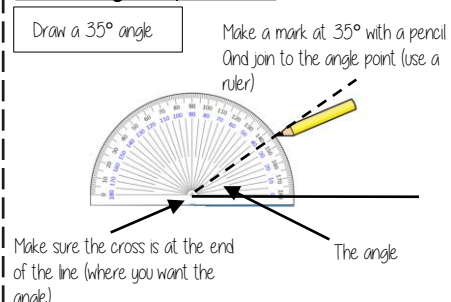
Classify angles



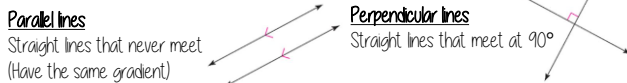
Measure angles to 180°



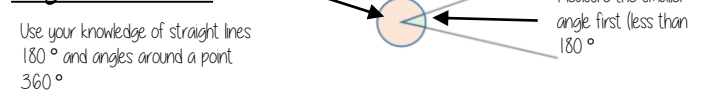
Draw angles up to 180°



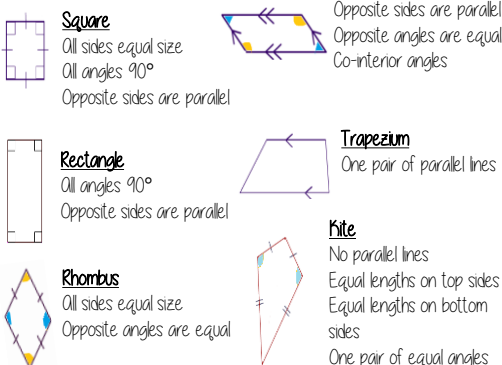
Parallel and Perpendicular lines



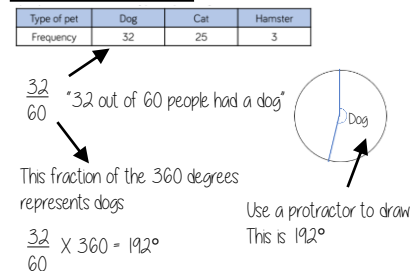
Angles over 180°



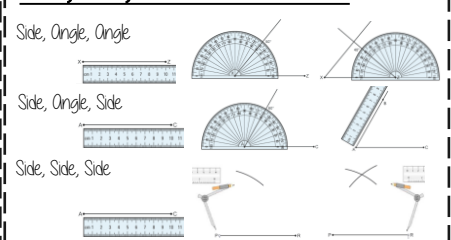
Properties of Quadrilaterals



Draw Pie Charts



SAS, SSS, ASA constructions



Polygons

3	- Triangle	5	- Pentagon	8	- Octagon
4	- Quadrilateral	6	- Hexagon	9	- Nonagon
		7	- Heptagon	10	- Decagon

If all the sides and angles are the same, it is a **regular** polygon

YEAR 7 — LINES AND ANGLES

Geometric reasoning

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand/use the sum of angles at a point
- Understand/use the sum of angles on a straight line
- Understand/use equality of vertically opposite angles
- Know and apply the sum of angles in a triangle
- Know and apply the sum of angles in a quadrilateral

Keywords

Vertically Opposite: angles formed when two or more straight lines cross at a point

Interior Angles: angles inside the shape

Sum: total, add all the interior angles together

Convex Quadrilateral: a four-sided polygon where every interior angle is less than 180°

Concave Quadrilateral: a four-sided polygon where one interior angle exceeds 180°

Polygon: A 2D shape made with straight lines

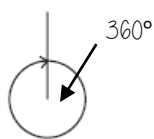
Scalene triangle: a triangle with all different sides and angles

Isosceles triangle: a triangle with two angles the same size and two angles the same size

Right-angled triangle: a triangle with a right angle

Sum of angles at a point

The sum of angles around a point is 360°



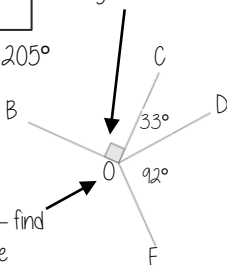
Find angle BOE

$$90^\circ + 33^\circ + 92^\circ = 205^\circ$$

$$360^\circ - 205^\circ$$

$$\text{BOE} = 155^\circ$$

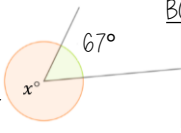
Angle notation — 90°



Angle notation — find this missing angle

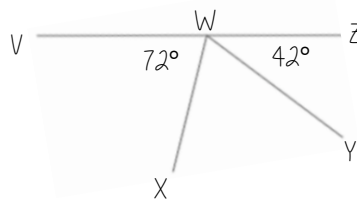
$$360^\circ - 67^\circ$$

$$= 293^\circ$$



Sum of angles on a straight line

Adjacent angles that share a common point on a line add up to 180°

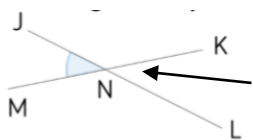


Find angle XWY

$$72^\circ + 42^\circ = 114^\circ$$

$$180^\circ - 114^\circ = 66^\circ$$

Vertically opposite angles

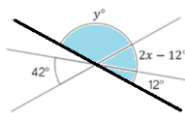


Angle JNM is vertically opposite to angle KNL

$$\text{JNM} = \text{KNL}$$

Vertically opposite angles are the same

Other angle rules still apply
Look for straight line sums and angles around a point



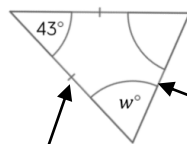
Form equations with information from diagrams

$$2x - 12 = 42$$

$$2x = 54$$

$$x = 27^\circ$$

Sum of angles in triangles



The two base angles will be the same size

Look at triangle notation
This indicates an isosceles triangle

$$\therefore 180 - 43 = 137$$

$$137 \div 2 = 68.5^\circ$$

A triangle can only have ONE right angle

Sum of interior angles in a triangle = 180°



Have a go!

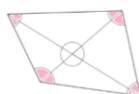
Tearing the corners from triangles forms a straight line which is therefore 180°

Sum of angles in quadrilaterals



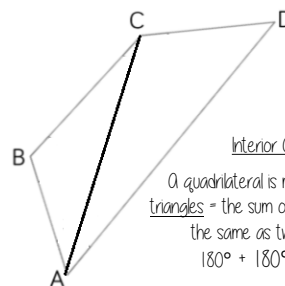
Convex Quadrilateral

Concave Quadrilateral



Interior angles are those that make up the perimeter (outline) of the shape

Sum of interior angles in a quadrilateral = 360°

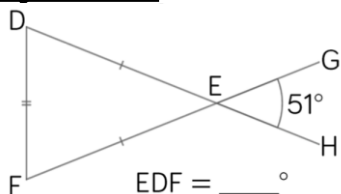


Interior Angles

A quadrilateral is made up of two triangles = the sum of interior angles is the same as two triangles
 $180^\circ + 180^\circ = 360^\circ$

Angle Problems

Split up the problem into chunks and explain your reasoning at each point using angle notation



1. Angle DEF = 51° because it is a vertically opposite angle DEF = GEH

2. Triangle DEF is isosceles (triangle notation) $\therefore$ EDF = EFD and the sum of interior angles is 180°
 $180^\circ - 51^\circ = 129^\circ$ $129^\circ \div 2 = 64.5^\circ$

3. Angle EDF = 64.5°

Keep working out clear and notes together

YEAR 7
KNOWLEDGE ORGANISERS



BLOCK: REASONING WITH NUMBER

Developing number sense

Sets and probability

Prime numbers and proof

YEAR 7 — REASONING WITH NUMBER

@whisto_maths

Developing number sense

What do I need to be able to do?

By the end of this unit you should be able to:

- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

Keywords

Commutative: changing the order of the operations does not change the result

Associative: when you add or multiply you can do so regardless of how the numbers are grouped

Dividend: the number being divided

Divisor: the number we divide by

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Equation: a mathematical statement that two things are equal

Quotient: the result of a division

Mental methods for addition/ subtraction

Addition is commutative



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

Mental methods for multiplication/ division

Multiplication is commutative



$$2 \times 4 = 4 \times 2$$

The order of multiplication does not change the result

Partitioning can help multiplication

$$\begin{aligned} 24 \times 6 &= 20 \times 6 + 4 \times 6 \\ &= 120 + 24 \\ &= 144 \end{aligned}$$

Division is not associative

Chunking the division can help $4000 \div 25$
"How many 25's in 100" then how many chunks of that in 4000.

Mental methods for decimals

Multiplying by a decimal < 1 will make the original value smaller e.g. $0.1 = \div 10$

Methods for multiplication 12×0.03

$$\begin{array}{l} 12 \times 3 = 36 \\ 12 \times 3 = 36 \\ 12 \times 0.3 = 3.6 \\ 12 \times 0.03 = 0.36 \end{array} \quad \begin{array}{l} 12 \times 3 = 36 \\ +10 \downarrow +100 \downarrow +1000 \downarrow \\ 12 \times 0.03 = 0.36 \end{array}$$

Methods for addition $2.3 + 2.4$

$$\begin{array}{l} 2 + 2 = 4 \\ 0.3 + 0.4 = 0.7 \\ 4 + 0.7 = 4.7 \end{array}$$

Methods for division $15 \div 0.05$

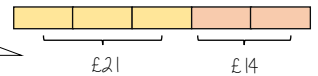
Multiply by powers of 10 until the divisor becomes an integer

$$\begin{array}{l} 1.5 \div 0.05 \\ \times 100 \downarrow \times 100 \downarrow \\ 150 \div 5 = 30 \end{array}$$

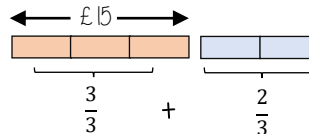
Mental methods for fractions

Use bar models where possible

I've spent $\frac{2}{5}$ of my money I have £21 left



How much did they have to begin with?



What is $\frac{5}{3}$ of £15?

Using factors to simplify calculations

$$30 \times 16$$

$$10 \times 3 \times 4 \times 4$$

$$10 \times 3 \times 2 \times 8$$

$$2 \times 5 \times 3 \times 2 \times 2 \times 2 \times 2$$

$$16 \times 10 \times 3$$

Multiplication is commutative
Factors can be multiplied in any order

Estimation

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

$$210 + 899 < 1200$$

This is true because even if both numbers were rounded up, they would reach $300 + 900$.

The correct estimation would be $200 + 900 = 1100$.

Number facts

Use

$$124 \times 5 = 620$$

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

$$620 \div 124 = 50$$

For division you must consider the impact of the divisor becoming smaller or bigger.

Smaller — the answer will be bigger
(It is being shared into less parts)
Bigger — the answer will be smaller
(It is being shared into more parts)

Algebraic facts

$$2a + 2b = 10$$

Everything $\times 2$

$$0.1a + 0.1b = 0.5$$

Everything $\div 10$

$$a + b = 5$$

Add 2 to the total

$$a + b + 2 = 7$$

The unknown quantity isn't changing but the variables change what is done to give the result

YEAR 7 — REASONING WITH NUMBER

Sets and probability

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale

Keywords

Set: collection of things

Element: each item in a set is called an element

Intersection: the overlapping part of a Venn diagram (AND $\cap$)

Union: two ellipses that join (OR $\cup$)

Mutually Exclusive: events that do not occur at the same time

Probability: likelihood of an event happening

Bias: a built-in error that makes all values wrong (unequal) by a certain amount, e.g. a weighted dice

Fair: there is zero bias, and all outcomes have an equal likelihood

Random: something happens by chance and is unable to be predicted

Identify and represent sets

The **universal set** has this symbol ξ — this means **EVERYTHING** in the Venn diagram is in this set

A set is a collection of things — you write sets inside curly brackets { }

$\xi = \{\text{the numbers between 1 and 50 inclusive}\}$

My sets can include every number between 1 and 50 including those numbers

$A = \{\text{Square numbers}\}$

$A = \{1, 4, 9, 16, 25, 36, 49\}$

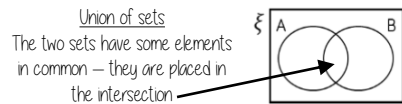
All the numbers in set A are square number and between 1 and 50

Interpret and create Venn diagrams



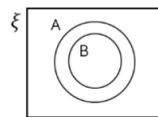
Mutually exclusive sets

The two sets have nothing in common
No overlap



Union of sets

The two sets have some elements in common — they are placed in the intersection



Subset

All of set B is also in Set A so the ellipse fits inside the set

The box

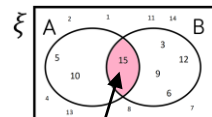
Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

Intersection of sets

Elements in the intersection are in set A AND set B

The notation for this is $A \cap B$

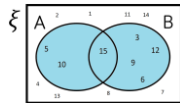
$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$
 $A = \{\text{Multiples of 5}\}$ $B = \{\text{Multiples of 3}\}$



The element in $A \cap B$ is 15

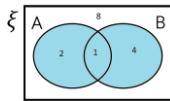
In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets



Elements in the union could be in set A OR set B

The notation for this is $A \cup B$



This Venn shows the **number of elements** in each set

$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$
 $A = \{\text{Multiples of 5}\}$ $B = \{\text{Multiples of 3}\}$

The elements in $A \cup B$ are
5, 10, 15, 3, 9, 6, 12

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15

Sample space — for single events



A sample space for rolling a six-sided dice is $S = \{1, 2, 3, 4, 5, 6\}$



A sample space for this spinner is $S = \{\text{Pink, Blue, Yellow}\}$

You only need to write each element once in a sample space diagram

- A Sample space represents a possible outcome from an event
- They can be interpreted in a variety of ways because they do not tell you the probability

Probability of a single event



Probability = $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

$P(\text{Blue}) = \frac{4}{10}$ ← There are 4 blue sectors
← There are 10 sectors overall
 $= \frac{2}{5}$

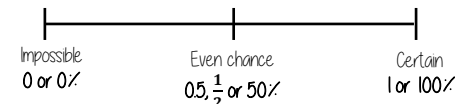
Probability notation
 $P(\text{event})$

Probability can be a fraction, decimal or percentage value

$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$

Probability is always a value between 0 and 1

The probability scale



The more likely an event the further up the probability it will be in comparison to another event
(It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes
So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Sum of probabilities

Probability is always a value between 0 and 1



The probability of getting a blue ball is $\frac{1}{5}$
∴ The probability of **NOT** getting a blue ball is $\frac{4}{5}$
The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$



YEAR 7 — REASONING WITH NUMBER

Prime numbers and Proof

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers

Factor: integers that multiply together to get another number.

Prime: an integer with only 2 factors

Conjecture: a statement that might be true (based on reasoning) but is not proven

Counterexample: a special type of example that disproves a statement

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

HCF: highest common factor (biggest factor two or more numbers share)

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

Non example of a multiple

45 is not a multiple of 3 because it is 3×15

Not an integer

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Factors

Arrays can help represent factors

5×2 or 2×5

Factors of 10
1, 2, 5, 10

10×1 or 1×10

Factors and expressions

$x \times x \times x \times x \times x$

The number itself is always a factor

Factors of $6x$

$6, x, 1, 6x, 2x, 3, 3x, 2$

$6x \times 1$ OR $6 \times x$

$x \times x$
 $x \times x$

$2x \times 3$

$x \times x \times x$
 $x \times x \times x$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number
The only even prime number

2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

odd even odd

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

Triangular numbers

Representations are useful — an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

Common factors are factors two or more numbers share

HCF — Highest common factor

HCF of 18 and 30

18 1, 2, 3, 6, 9, 18

30 1, 2, 3, 5, 6, 10, 15, 30

Common factors
(factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM — Lowest common multiple

LCM of 9 and 12

LCM = 36

The first time their multiples match

9 9, 18, 27, 36, 45, 54

12 12, 24, 36, 48, 60

9 12 27 36 45
12 24 36 48

Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$ and $\frac{7}{10}$

Conjectures and counterexamples

Conjecture

1, 2, 4...

The numbers in the sequence are doubling each time.

Counterexamples



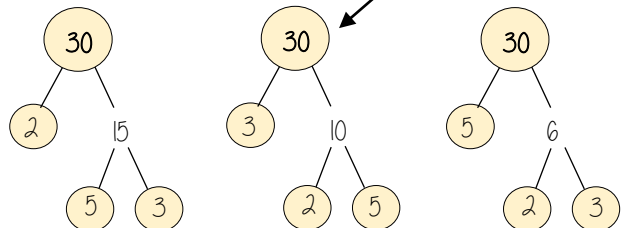
This sequence isn't doubling it is adding 2 each time

A pattern that is noticed for many cases

Only one counterexample is needed to disprove a conjecture

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

e.g 60 30×2 $2 \times 3 \times 5 \times 2$
150 30×5 $2 \times 3 \times 5 \times 5$